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WROUGHT IRON BRIDGES AND ROOFS.

Plate XIV.

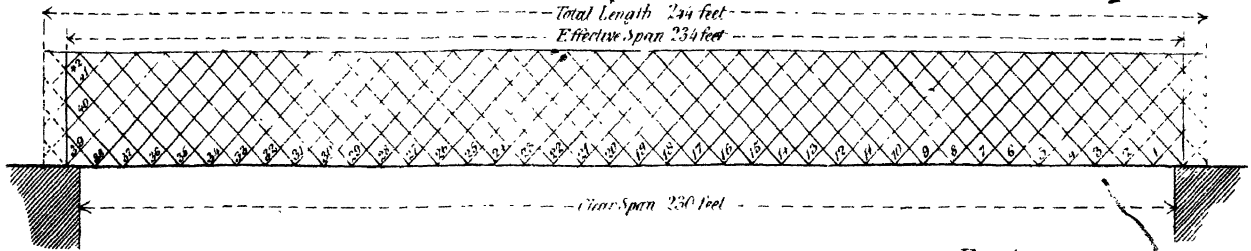


Fig. 1

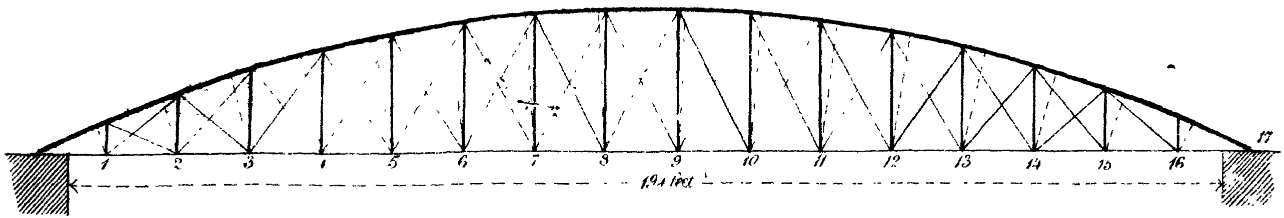


Fig. 2.

The dotted lines in Fig. 2 are perpendiculars on the arched Boom —
The thrusts are ascertained at the points where these perpendiculars

# WROUGHT IRON

# BRIDGES AND ROOFS.

*LECTURES DELIVERED AT THE ROYAL ENGINEER ESTABLISHMENT, CHATHAM.*

*WITH EXAMPLES OF THE CALCULATION OF STRESS IN GIRDERS AND ROOF TRUSSES BY GRAPHIC AND ALGEBRAIC METHODS.*

BY

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ASSOCIATE INST. C. E.

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1869.

TO

WILLIAM FAIRBAIRN, ESQ., C.E.,

LL.D., F.R.S., F.G.S.,

CORRESPONDING MEMBER OF THE INSTITUTE OF FRANCE, AND THE ROYAL ACADEMY OF TURIN; CHEVALIER OF THE LEGION OF HONOUR; &c., &c.

DEAR SIR,

It has been my good fortune to have assisted you in some of the many researches by which you have exemplified the value of the scientific study of practical questions, and the importance of experimental data for useful theorising. I am glad to be allowed to inscribe to you these Lectures, treating of structures which are but various modifications and developments of those first great wrought-iron girder bridges, which owe so many of their essential features to your sagacity.

Believe me,

With much esteem,

Yours very truly,

W. CAWTHORNE UNWIN.

*Empirici, formicæ more, congerunt tantum et utuntur; Rationa aranearum more, telas ex se conficiunt: apis vero ratio media est, q materiam ex floribus horti et agri elicit, sed tamen eam propria facult vertit et digerit. Neque absimile philosophiæ verum opificium est; q nec mentis viribus tantum aut præcipue nititur, neque ex historia natur et mechanicis experimentis præbitam materiam, in memoria integram, in intellectu mutatam et subactam, reponit.*

BACON. NOV. ORG. I. APH.

# PREFACE.

THESE Lectures were delivered in the spring of 1868, before the officers of the Royal Engineers, under instruction at Chatham, and were afterwards printed at the press of that establishment for private circulation. They proved useful to those to whom they were originally addressed, and it is hoped that in their present form they may be of service to a wider circle of readers. They have been carefully revised, subdivided somewhat differently to secure a more scientific arrangement of the matters discussed, and here and there investigations have been added which could hardly have been rendered intelligible in the brief space allowed for their delivery. The three examples given in the Appendix were prepared as illustrations of the application of the principles laid down in the Lectures. Most Engineers who have been engaged in the construction of bridges will be familiar with such calculations. But for others there is a gap between the enumeration of formulæ and methods in ordinary treatises and their application in actual practice, which such examples may help to bridge over. They have, therefore, been reprinted both as illustrations of the manner of setting about calculations on structures of this kind, and to serve as a means of reference, in the order required in carrying out such calculations, to the principles previously laid down.

With books of a practical character, describing executed works, our technical literature is well provided. Nor is it wanting in admirable theoretical treatises, in which the application of such truths as can be expressed in mathematical forms is developed by analysis, so as to comprehend more or less completely all possible cases. The present work belongs to an intermediate class, treating of the application of theoretical principles only so far as they relate to a special order of structures, and are necessary for dealing with the forms and circumstances of ordinary practice. Without hesitating to avail

himself of mathematics when really necessary, the author felt it desirable to restrict the use of symbolic expressions as much as possible. On the other hand, whilst aiming to be practical, it appeared to him that practical information could only be usefully given in oral lectures, so far as it could be made to illustrate general principles. Without pretending to have been always successful and consistent in carrying out these views, they will be found to explain the form which this treatise has taken.

Much is said at the present time of the necessity of *technical education*, professorial teaching being very often all that is implied in the expression. Important as such teaching is, and much as it is needed in this country, it would be well to remember that its function is stringently limited. The multifarious details which go to make the knowledge of a practical man print themselves on his memory by association with the other facts of his daily life, with a force and vividness unattainable in any other way. Technical knowledge of this kind cannot be communicated orally in the lecture-room. The teacher of a technical subject must in general restrict himself to such knowledge as can be classified, comprehended in laws, and which has already become part of science in the strict sense. The mere description of details of operations and of executed works is of comparatively small use, except to those who have been actually engaged in the like.

What Bacon has said of science generally, in his famous fable of the ants, the spiders, and the bees, is especially true of applied science. The ants, says Bacon, only heap up and use what they collect; the spiders spin webs out of themselves; the method of the bee is between these, it extracts matter from the garden and the field, but converts and digests it by its own faculty. The true labour of the teacher of applied science does not differ from that of the bee, for he must not rely simply on the powers of the mind, nor simply accumulate undigested in the memory the raw facts of professional experience, but must endeavour to store such facts in the understanding, after first modifying and subduing them.

In designing any structure with due regard to economy and strength, three inquiries present themselves:—The intensity of stress the material will safely bear, the forces to which the structure will be subjected, and the actual stresses to which those forces will give rise. These

three inquiries are of co-ordinate importance, although very often the third only receives careful consideration. Instances will be within the experience of all who have had to do with engineering works, where most elaborate calculations of stress have been founded on a guess as to the forces to be sustained, and the structure afterwards designed to a traditional factor of safety. And in many cases the determination of the probable maximum load and of the proper limiting stress is much more perplexing than the calculation of the stresses. It is desirable in a special treatise, like the present, that these three questions should receive equal attention, and the author has endeavoured to point out, how the estimate of the load may be made with an accuracy corresponding to that with which the stress calculations are ordinarily carried out, and on what principles the safe limit of stress should be determined.

In no case, perhaps, has more useless ingenuity been expended than in the production of cumbrous trigonometrical formulæ for the stresses in roofs,—formulæ applicable only to a condition of uniform vertical loading, and useless when the actual nature of the forces to which a roof is subjected is taken into account. The author trusts that in the discussion of the effect of wind pressure some real advance has been made, and that the examples of the beautiful graphic method of Professor Clerk Maxwell may prove of essential service to the designer of roofs. Graphic methods have hardly yet received, at all events in this country, the attention they deserve. In cases like those of roofs they are most powerful, and their principle once thoroughly mastered, they are most facile and accurate in application.

In the first Lecture the general principles applicable to the determination of the stress at any section of a straight beam, in whatever manner loaded, are discussed, and geometrical representations of the formulæ are given in all the cases ordinarily occurring in the construction of wrought-iron girders. Practically all wrought-iron girders are flanged girders, and in determining the stress from the shearing force and bending moment, the restriction of the investigation to this form of section introduces great simplicity.

In the second Lecture the principles by which the limit of intensity of stress should be fixed are discussed. Formulæ are given by which the approximate weight of a girder may be determined from the load

which it is required to carry. The load to be allowed for in road and railway bridges is examined, both in those cases in which the assumption of uniform distribution leads to no practical error, and in those in which it is necessary to take into account the concentration of the load at isolated points. The curves of maximum bending moment due to the passage of a locomotive engine over girders of short span have been drawn, the author believes, for the first time.

In the third and fourth Lectures the proportions and construction of plate web and braced girders are discussed, and in the latter the special methods of estimating the stresses applicable to braced girders. The author has always used Mr. Latham's method for parallel braced girders, the principle of which is here deduced from the simple consideration of the shear and bending moment at a section. Mr. Latham was the first to abandon completely the formulæ for breaking weight and to proportion girders simply to the actual stresses due to the load, and his work is still probably the best treatise in our language on wrought-iron bridges. Lastly, applications are given of graphic methods to parallel and bowstring girders.

In the fifth Lecture the subject of rivetted joints is discussed. Although frequently experimented on, perhaps no experimental inquiry would bear more immediate fruit than a new and thorough investigation of the strength of rivetted joints, if undertaken by anyone competent to analyse the effect of the inequality of the distribution of stress due to various arrangements of rivetting.

In the Lecture on roofs the forces to be sustained are discussed, various types of trussed roof are examined, and the beautiful graphic method of Professor Rankine and Professor Clerk Maxwell is applied to the determination of the stresses both with symmetrical and unsymmetrical loads, in examples of each type of roof. The subject of arched roofs has been omitted from a feeling that the mere description of those structures was not required, unless at the same time the theory of their construction could have been given. On examination the author found that to do so would have involved formulæ of a complexity out of harmony with an elementary treatise like the present. The ordinary formulæ given for arched roofs depend on arbitrary assumptions, as for instance the hinging of the arch at the springing and crown, which render them very unsatisfactory, if not useless.

## CONTENTS.

### LECTURE I.—ON THE DISTRIBUTION OF STRESS IN BEAMS.



### LECTURE II.—ON THE INTENSITY OF STRESS IN BRIDGES AND ON ESTIMATING THE LOAD.



### LECTURE III.—ON PLATE WEB BRIDGES.

### LECTURE IV.—ON BRACED GIRDERS.



### LECTURE V.—ON RIVETTED AND OTHER JOINTS.



### LECTURE VI.—ON ROOFS.



### APPENDIX.—EXAMPLES OF CALCULATION OF STRESS.



### PLATES.

# ERRATA.

PAGE

6, Fig. 2, *insert* A *above the right*, B *above the left abutment.*

8, *lines* 9, 10, and 11, *for* $\frac{6}{19}$, $\frac{86}{95}$ and $\frac{74}{95}$, *read* $\frac{15}{38}$, $\frac{43}{38}$ *and* $\frac{37}{38}$ *respectively.*

19, line 9, *read* the corresponding vertical ordinate.

42, line 7 from foot, *after* $x$, *add* due to a series of loads each equal to P.

73, line 6, *after* vanishes, *add* or becomes a minimum.

76, formula (3.), *read* t = wl ÷ n.

77, line 16, *after* material, *add* neglecting joint covers.

100, line 10 from foot, *for* working load, *read* working live load.

109, line 7, *for* ·78s, *read* ·78s₁.

110, line 3, *read* the thickness $t$ of the separate plates.

120, last line, *insert* 90 *in first column of Table.*

130, line 6 from foot, *for* Plate VII., *read* Plate VIII.

132, line 4 from foot, *for* 81, *read* 80.

„ lines 5 and 8 from foot, *for* 82, *read* 81.

141, line 3, *after* and, *add* these lines.

143, line 20, *for* load, *read* loads.

„ line 30, *for* point, *read* joint.

163, line 3, *for* increases, *read* decreases.

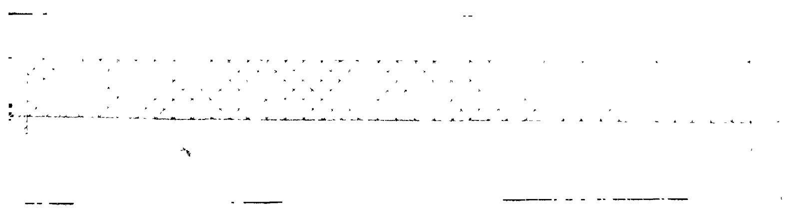
Plate IX., Fig. 2, *for* 2 — 4 *read* 6 — 7.

„ „ „ 4 — 6 „ 2 — 4.

„ „ „ 6 — 7 „ 1 — 2.

„ „ „ 1 — 2 „ 4 — 6.

Plate XIV., Fig. 1, has been drawn with the bracing at 44° instead of 47°, which gives one system of triangulation more than is assumed in the calculations. The following figure is correct.



theory of beams, that is, a theory in which all the molecular actions in a bent bar should be comprehended under mathematical laws. Nevertheless, within a defined limit, theory affords a means of determining, with almost absolute

# LECTURE I.

## *ON THE DISTRIBUTION OF STRESS IN BEAMS.*

1. It is proposed in these Lectures to lay down, so far as may be possible within the limits assigned to them, such knowledge of principles and of precedents, as ought to be before the mind of an Engineer sitting down to a blank sheet of paper to design a bridge or a roof. Such an Engineer requires a knowledge partly theoretical and partly practical: from theory he should have learnt how and in what proportions to dispose his material, to resist given forces; from experience he should have ascertained to what forces his structure is likely to be subjected, to what extent his material is to be trusted, and in what forms it can be most easily obtained and most economically wrought.

From the commencement of the modern epoch of Engineering,—that epoch marked by the great extension of the use of iron in construction,—the laws of transverse strain have been repeatedly investigated both by Mathematicians and Engineers, and the result is, that theory has been brought into a closer relation with practice in the construction of roofs and bridges, than in almost any other department of Engineering, and is applied with the utmost certainty to structures of the greatest magnitude and importance. It is true that we are still far from having a complete physical theory of beams, that is, a theory in which all the molecular actions in a bent bar should be comprehended under mathematical laws. Nevertheless, within a defined limit, theory affords a means of determining, with almost absolute exact-

ness, the stress at any part of a transversely loaded bar or beam, the condition of its applicability being that the stress should nowhere exceed that at which the elasticity of the material remains sensibly perfect.

It is necessary to call attention to this point at the outset, because much confusion has arisen from the fact that, whilst all general formulæ for beams are founded on the consideration of a perfectly elastic material, and possess no theoretical validity in other conditions, almost all our useful data on the strength of materials are derived from experiments made at the breaking point, when the condition of perfect elasticity is no longer fulfilled. Iron remains perfectly elastic only for a definite range of stress. Using the convenient distinction, introduced by Professor Rankine and others, between *stress* and *strain*,—taking stress to mean the force applied to a material body, and strain the alteration of volume or figure which results from the application of stress,—then the strain is sensibly proportional to the stress for a range of only about one-third of the whole stress which may be applied before rupture ensues, if the bar has not previously been strained, and for a range of perhaps two-thirds of the breaking stress if the bar has been previously loaded with nearly the whole breaking weight. But in either case with loads near the breaking weight the strain is not proportional to the stress, and the condition of perfect elasticity is not fulfilled. Hence laws derived from the consideration of a perfectly elastic material will not give accurately the ultimate resistance of structures of wrought iron.

2. Equations, of which the form is derived from the assumption of perfect elasticity, and the co-efficients from actual experiments on the breaking weight of bars, are found to express, with considerable accuracy, the ultimate strength in terms of the dimensions for bars of similar form, and for those limits of variation in the dimensions within which experiment is possible. And few formulæ have rendered such essential service to Engineering, as the empirical formulæ of this kind introduced by Professor Hodgkinson for cast, and by Dr. Fairbairn for wrought-iron beams. But, at

best, such formulæ represent, with more or less accuracy, the results of a limited number of experiments, and are unreliable when applied to cases far beyond the range of possible experiment. If the Engineer were restricted to formulæ of this kind, there would always remain an element of uncertainty in his calculations.

Now, as a matter of fact, great girder constructions are not submitted in practice to strains greater than those at which the elasticity of the iron remains sensibly perfect. And experience shows that they could not be permanently safe if strained beyond that limit. Hence, paradoxical as it may seem, the ultimate strength of great girder constructions is probably not accurately known, and is certainly not required to be known, in order to be assured of their stability. Probably, in the future of Engineering, empirical formulæ for the ultimate strength will be more and more discarded. It is, to say the least, a roundabout proceeding to calculate the breaking weight, and then to divide it by an assumed factor of safety to find the safe load. The safety of the structure will be more directly assured by estimating the actual stresses, and by providing such an amount of material that the intensity of the stress shall nowhere exceed three, or four, or five tons per square inch, or whatever limiting stress is shown by experience to be suitable for the material. In this way the stress on any member of a structure can be theoretically ascertained without any illicit assumption.

The first requisite, therefore, in designing a structure is a thorough knowledge of the distribution of stress, due to the various modes of loading, which may occur in practice. That the Engineer may not be merely a slave to formulæ, he should attain to a clear mental image of the distribution of stress, and should mould his material almost as a sculptor cuts the marble to an ideal form pre-existent in his mind. To render the structure secure, a quantity of material proportional to the stress must be provided in every part; and economy requires that the material should nowhere exceed that proportion. Uniform stress is, therefore, the condition of a perfect structure; understanding, however, by uniformity

of stress, not that the structure should ever in actual circumstances be uniformly strained, but that there should be uniformity in the maximum limit of the intensity of the stresses on the parts, under all the varying conditions of the life of the structure.

## Distribution of Stress with Transverse Load.

3. If to a bar, fixed at one end or resting on two supports, a load be applied, the bar bends and comes to a position of equilibrium. Then, given the loads, the simple laws of statics furnish equations to the supporting forces.

The first point to notice is the character of the load. The external load may be either concentrated at one or more points, or uniformly distributed over the length of the beam. In either case it may be a dead or permanent load, or a live load more or less suddenly and more or less frequently imposed and removed. Finally, a live load frequently assumes the form of a rolling or travelling load, gradually loading the structure from one end. A railway train is a load of this kind, which forms an important special case. It is sufficiently accurate for practical purposes to assume that the weight of the structure itself is a uniformly distributed dead load.

In the next place the equilibrium between the loading and supporting forces is not established directly, but through the intermediation of forces transmitted through the beam which develop molecular forces in the beam: and, consequently, if we take any section of a bent bar, then all the external loading and supporting forces on one side of that section are in equilibrium with the molecular forces at the section.

In determining the stresses at a section of a beam, therefore, it is only necessary to consider the external forces on one side of the section.

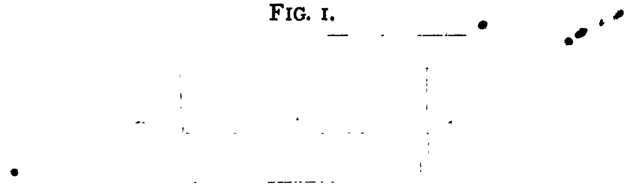
4. *Determination of the supporting forces.* If a beam is fixed at one end and free at the other, it must be supported by a couple, acting on the part which is *encastré*, whose moment is equal and opposite to the sum of the moments of the loading forces about the point on which the beam is supported.

If the beam is supported at both ends and loaded between

the supports, by forces in one plane, then resolving the forces in the direction of and perpendicular to the axis of the beam, the sum of the components of the loading and supporting forces parallel to the beam must be *zero*; of those perpendicular to the beam must be *zero*; and the sum of the moments of all the external forces about any point in the beam must be *zero*.

Let $AB$ be a beam supported at both ends, and loaded at

FIG. I.



$C$ by a force $P$ making an angle $a$ with the axis of the beam. The component of $P$ parallel to the beam is $P \cos a$, and there will equally be equilibrium whether the component of the supporting forces $-P \cos a$ is applied at $A$ or $B$ or divided between them. If the beam is fixed horizontally at one end as at $A$ and perfectly free to move at $B$, the force $-P \cos A$ will be applied at $A$, and the reaction at $B$ will be vertical. In other cases there will be an ambiguity, and the beam should be strong enough to resist the stresses in the extreme cases to which the shifting of the horizontal reaction gives rise. Thus, if the beam be fixed at $A$ and rest on rollers at $B$, the maximum horizontal resistance at $B$ will be the frictional resistance to motion at that point. Let $s_1$ be the vertical reaction at $B$. Then the frictional resistance at $B$ will be represented by $\pm fs_1$ where $f$ is a coefficient depending on the nature of the material, number, and form of the rollers. And the beam should be made strong enough to resist the stresses due to $-(P \cos a + fs_1)$ at $A$ and $+fs_1$ at $B$, and those due to $-(P \cos a - fs_1)$ at $A$ and $-fs_1$ at $B$; positive forces being those acting in the direction $AB$, and negative those in the direction $BA$. These considerations are of importance in determining the stresses on roofs of large span.

5. In the case of bridges, the loading and supporting forces are all vertical. Let $AB$ be a beam supported at $A$ and $B$, and loaded by any number of forces $p_1$, $p_2$, . . . acting at distances $x_1$, $x_2$, . . . from $B$. Then the supporting force at $A$,

$$s_1 = \frac{p_1 x_1 + p_2 x_2 \ldots}{l} = \Sigma \frac{px}{l}.$$

And that at B,

$$s_2 = \frac{p_1 (l - x_1) + p_2 (l - x_2) \ldots}{l} = \Sigma \frac{p\,(l-x)}{l}.$$

FIG. 2.

The following construction may sometimes be useful in determining the supporting forces. On two rectangular axes $Ox$, $Oy$, take $Oa$, $Ob$, . . . equal, on any scale, to the distances $x_1$, $x_2$, . . . of the points of application of the forces from one abutment $B$; and $Og$ to the span $l$. Take on any scale $Oc$, $Od$, . . . equal to the loads $p_1$, $p_2$, . . . complete the rectangles $ac$, $bd$, and produce the sides; through $g$ draw $gef$ parallel to $Ox$, cutting the sides produced of the rectangles in $e$, $f$; through $e$, $f$, and $O$ draw $eh$, $fk$, cutting the corresponding sides produced of the rectangles in $h$, $k$.

FIG. 3.

Then the reaction at $A$ is represented by the sum of the intercepts $ah$, $bk$, on the same scale as that on which $Oc$, $Od$, represent the forces. The construction is applicable to any number of forces.

If the beam is uniformly loaded with $w$ tons per unit of span, then the reaction at either abutment is half the total distributed load, or

$$s_1 = s_2 = \tfrac{1}{2}\, w\, l \text{ tons.}$$

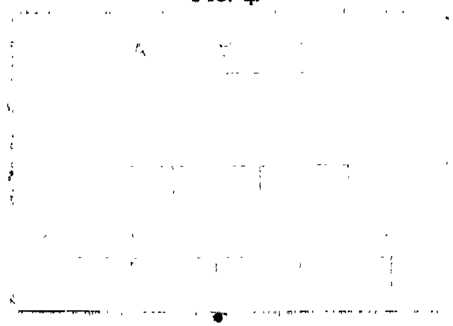
6. If the beam is supported at more than two points, the supporting forces can only be determined by the joint consideration of the loads and the molecular forces, and in its general form the solution of the problem is very complex. There are, however, certain simple cases of importance in the determination of the stresses in roofs for which the supporting forces may be given. The common rafter and purlin of a roof, for instance, are generally beams of uniform section, under uniformly distributed loads, or at least approximate to that condition, and are supported at three or more points. In such a case we require to know the reactions at the points of support, both in estimating the forces transmitted to the roof principals, and in determining the stresses on the rafters and purlins themselves.

A uniform prismatic beam, supported by piers at equidistant points, at the same level or in one straight line, and uniformly loaded by forces amounting to $w$ units of weight per unit of length, will give rise to the following reactions at the supports:—

(1) Beam of two equal spans on three supports, the total load on one span of the beam being $wl$—

$$\text{Reaction at } A \text{ and } C = \tfrac{3}{8}\, wl.$$
$$\text{,, \quad ,, \quad } B = \tfrac{5}{4}\, wl.$$

FIG. 4.



(2) Beam of three equal spans on four supports—

Reaction at $A$ and $D = \frac{4}{10}\, wl.$

" " $B$ and $C = \frac{11}{10}\, wl.$

(3) Beam of four equal spans on five supports—

Reaction at $A$ and $E = \frac{11}{28}\, wl.$

" " $B$ and $D = \frac{8}{7}\, wl.$

" " $C = \frac{13}{14}\, wl.$

(4) Beam of five equal spans on six supports—

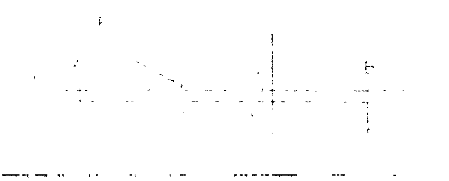
Reaction at 1st and 6th $= \frac{6}{19}\, wl.$

" " 2nd and 5th $= \frac{86}{95}\, wl.$

" " 3rd and 4th $= \frac{74}{95}\, wl.$

7. *Molecular forces at a section.* Let $AB$ (Fig. 5) represent a beam bent by transverse forces, and let $cd$ be any section at which it is desired to ascertain the stresses. Take a pair of rectangular axes $Ox$, $Oy$, one parallel, the other perpendicular to the section, and having the origin at the centre of gravity of the section, then the molecular forces at $cd$ must be in equilibrium with the external forces on either side of the section. If $-P$ represent one of these forces acting at

FIG. 5.

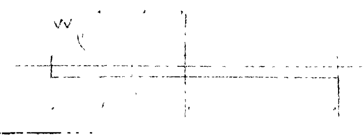


angle $\alpha$ with the axis $Ox$ it may be resolved into $-P\cos\alpha$ perpendicular to the section, and $-P\sin\alpha$ parallel to the section. The force $-P\cos\alpha$, if transmitted to the abutment $B$, produces at the section a direct tension or compression (in the case represented a compression) known as the longitudinal stress. The force $-P\sin\alpha$ tends to cause the part $OA$ of the beam to slide vertically downwards relatively to $OB$, and is in equilibrium at the section with a tangential stress due to the resistance of the material to shearing or sliding. The force $-P\sin\alpha$, and the equal and opposite

resistance to shearing $P \sin \alpha$, at the section, form a couple tending to cause the part $OA$ of the beam to revolve about an axis through $O$, whose moment, called the bending moment, is $P \sin \alpha . Oe$. The bending moment developes at the section equal tensions and compressions, forming a couple whose moment is equal and opposite to the bending moment, and is called the moment of resistance to flexure. The whole stress on $od$ is the algebraic sum of the separate stresses due to all the loading and supporting forces between $O$ and $A$.

8. In the case of a girder bridge all the loading forces are vertical, and there is no direct longitudinal stress in the sense explained above. Let $P_1$, $P_2$ be the reactions at the abutments; $x_1$, $-x_2$ the distances of the abutment from the origin; $x'$ the distance of any load $-W$ from the origin; $x$

FIG. 6.



the distance from the origin of a section at which the stresses are required. Then the shearing force on a section at $x$ due to the reaction of the abutment at $x_1$ is $P_1$, and that due to the load $-W$ is $-W$; so that putting $-\Sigma_x^{x_1} W$ for the sum of the loads between the section and the abutment at $x_1$, the shearing force on the section is

$$F = P_1 - \Sigma_x^{x_1} W.$$

The bending moment of the reaction at the abutment is $(x_1 - x) P_1$; that of the load $-W$ is $-(x' - x) W$, so that the whole bending moment at the section is

$$M = (x_1 - x) P_1 - \Sigma_x^{x_1} (x' - x) W.$$

If the dimensions are taken in feet, and the forces in tons, the shearing force will be in tons, and the bending moment in foot-tons. These formulæ are perfectly general, and put into words amount simply to this:—

(1) The shearing force is the algebraic sum of all the external forces on one side of the section; forces acting

upwards being considered positive, and those acting downwards negative.

(2) The bending moment is the algebraic sum of the products of the external forces on one side of the section, and their distances from the section.

The nature of the forces acting at a section of a transversely loaded beam may be illustrated by supposing such a section actually made, and considering how the disconnected end may be maintained in place. Let $AB$ be such a beam, cut through at $CD$. The tendency of $AD$ to revolve under the action the load will be resisted if a strut be interposed at $D$, and the link of a chain at $C$; there will then be a tension $t$ at $C$, and an equal compression $c$ at $D$, forming a couple whose moment is equal to the bending moment, so that

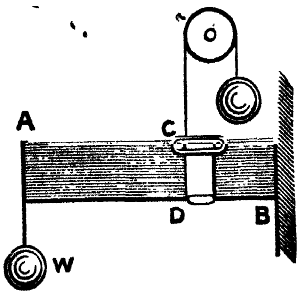



FIG. 7.

$$t.CD = c.DC = W.AC;$$

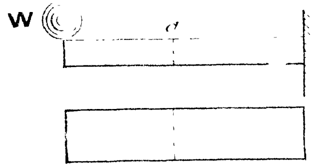
but in order that there may be equilibrium, a force parallel and opposite to the load must be introduced at $C$ equivalent to the resistance to shearing. This may be effected by a cord passing over a pulley, and carrying a weight equal to $W$.

9. The formulæ given above apply to all cases in which the directions of the loads are perpendicular to the axis of the beam. Their use may be illustrated by taking a series of cases, such as ordinarily occur in practice; and their results may be exhibited graphically by figures, the abscissæ of which correspond to the distances measured along the beam and the ordinates to the shearing force, or bending moments at those points. In no other way can the distribution of stress be so clearly realised, or so easily remembered; and as the graphic representation can be constructed with very slight use of formulæ it is often useful in designing beams and girders, as a substitute for detailed calculations.

## GRAPHIC REPRESENTATION OF SHEARING FORCE.

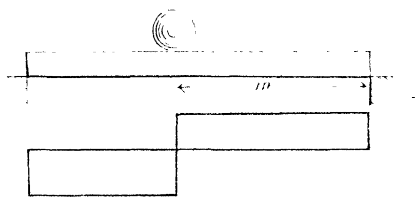
10. *Beam supported at one end and loaded at the other; weight of beam neglected.* For every section of a beam loaded at one end and supported at the other, the only external force on the free side of the section is the load $-W$. The shearing force will therefore be constant for every point of the beam and equal to $-W$. Such a distribution of shearing force is represented by a rectangle, the length of which corresponds with the projecting length of the beam, and the breadth with the load $-W$.

FIG. 8.




11. *Beam supported at both ends, and loaded at intermediate points; weight of beam neglected.* If a beam carry a load $-W$ at a point whose distance from the right abutment is $m$, the reaction at that abutment is $\frac{l-m}{l}W$. From the right abutment to the load, the shearing force is equal to that reaction and positive on the side of a section towards that abutment. From the load to the other abutment the shearing force is $\frac{l-m}{l}W - W$, or $-\frac{m}{l}W$, equal and opposite to the reaction at the left abutment, and negative on the side of a section towards the right abutment. The distribution of shearing force is given by two rectangles, the breadths of which are equal to the reactions at the abutments.

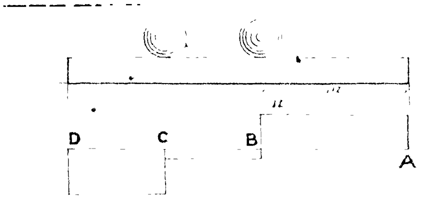
FIG. 9.



Suppose the same beam loaded with two weights, $W_1$, $W_2$, at points distant $m$ and $n$ from $A$. The reaction at $A$ is

$$P_1 = \frac{l-m}{l} W_1 + \frac{l-n}{l} W_2.$$

FIG. 10.




From $A$ to $B$ the shearing force on the side of a section towards $A$ is $P_1$; from $B$ to $C$ it is $P_1 - W_1$; from $C$ to $D$ it is $P_1 - W_1 - W_2$, or

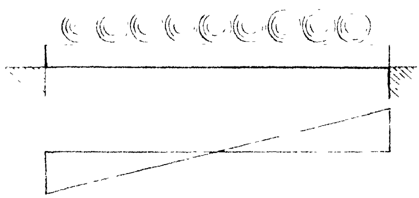
$$-\frac{m}{l} W_1 - \frac{n}{l} W_2,$$

equal and opposite to the reaction at $D$. The distribution of shearing force is represented by three rectangles corresponding to the segments into which the loads divide the beam, and the breadths of which are the reactions at the abutments, and the difference of those reactions.

When the weights are equal and equally distant from the abutments, an interesting case arises, for then the shearing force on the part of the beam between the weights vanishes, and the beam is there subject to a bending moment only.

12. *Beam uniformly loaded.* In this case, the load being $w$ per unit of span, the re-action at each abutment is $\frac{wl}{2}$. The shearing force beginning at $A$ is $\frac{wl}{2}$, diminishing uniformly to zero at the centre, and to $-\frac{wl}{2}$ at the further abutment; so

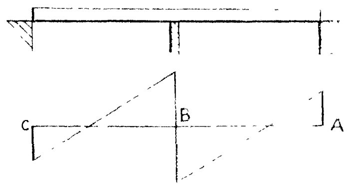
FIG. 11.



that the distribution of the shearing force is represented by two

triangles. This is a case in which the graphic method may be useful in designing girders. Taking any line to represent the span, and setting up an ordinate at one end, equal on any scale to half the distributed load, and at the other an equal ordinate measured negatively, and joining the extremities, the breadth of the diagram at any point is the shearing force corresponding to that point of the span.

FIG. 12.

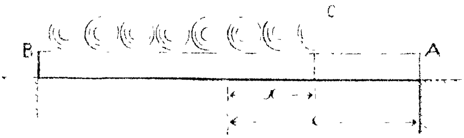



13. *Uniform beam uniformly loaded on three supports at the same level.* In this case the load being $w$ per unit of span, we already know, (§6,) that the reaction at the three supports is $\frac{3}{8}\,wl$, $\frac{5}{4}\,wl$, and $\frac{3}{8}\,wl$ respectively. At $A$ the shearing force is $\frac{3}{8}\,wl$; immediately on the right of $B$ the shearing force is $\frac{3}{8}\,wl - wl$, or $-\frac{5}{8}\,wl$; on the left of $B$ it is $\frac{3}{8}\,wl - wl + \frac{5}{4}\,wl = \frac{5}{8}\,wl$; at $C$ it is $(\frac{3}{8} - 1 + \frac{5}{4} - 1)\,wl = -\frac{3}{8}\,wl$. If on a line representing the two spans we set up ordinates equal to these values of the shearing force, and join the extremities, we shall get four triangles representing the distribution of shearing force, the breadth at any point representing the shearing force at that point.

14. *Beam with travelling load.* An extremely important special case, because it occurs in all railway bridges, is that in which the live load is a travelling load, gradually loading the structure from end to end. The peculiarity in this case is, that the shearing force so generated at each section increases to a maximum as the load approaches the section from the further abutment, and diminishes again as it passes the section and completes the loading of the bridge. In other words, the maximum shearing stress at a section occurs, when the front of the train or travelling load reaches that section, so that the longer of the two segments into which the section divides the beam is loaded and the shorter unloaded.

Suppose a beam (Fig. 13) subject to a travelling load, and let it be required to find the position of the load which gives the maximum shearing force at the point $C$. Suppose the longer

FIG. 13.



segment loaded and the shorter unloaded then the shearing force at $C$ is a maximum and equal to the reaction at $A$. For if weight be removed between $C$ and $B$, the reaction at $A$ is diminished, and the shearing force at $C$ is diminished by the same amount; and if weight is added between $C$ and $A$, the reaction at $A$ acting upwards is increased by part of the weight so added, but the shearing force at $C$, which is equal to the resultant of the forces between $A$ and $C$, is diminished by the whole of the weight so added, acting downwards or in the opposite direction to the reaction at $A$. In all railway bridges, therefore, it is necessary to make a special calculation of the shearing force due to partial loading.

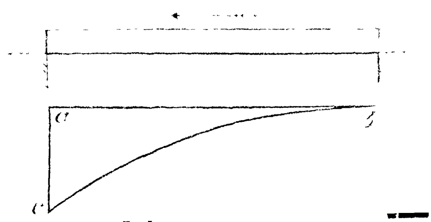
Let $w$ be the uniform live load per unit of span, $c$ the half span, $x$ the distance of the section at which the stress is required from the centre of the bridge. Then the length of the longer or loaded segment is $c + x$; the load on it is $w (c + x)$; the distance of the centre of gravity of the load from $B$ is $\frac{c+x}{2}$; the reaction at $A$ is $w\,(c + x)\,\frac{c+x}{2} \cdot \frac{1}{2c} = w\,\frac{(c+x)^2}{4c}$ which is also the maximum shearing force at $C$ due to the live load. If there is also a uniform dead load $w'$ per unit of span, the whole shearing force at $C$ is :—

$$F = w'\,x + w\,\frac{(c+x)^2}{4c}.$$

The excess of the shearing force due to partial over that due to complete loading is obviously nothing at the abutments, where the two cases are identical; at the centre it amounts to one-eighth of the total uniform live load.

15. If we take as before vertical ordinates on a line representing the span to represent the successive maxima of the shearing force at the various points, the curve through their extremities will evidently be a parabola with its vertex at the

FIG. 14.

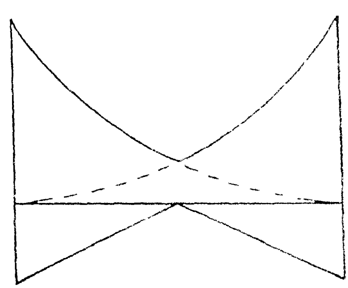


abutment at which the travelling load arrives, and its axis perpendicular to the beam. (Fig. 14.)

Now the shearing force at the abutment is half the total live load. Hence, by taking *a c*, equal on any scale to half the live load, the parabola *b c* may easily be drawn by means of a table of parabolic ordinates. Or if we so arrange the scales for the weights and span, that *a c* does not exceed one-fourth *a b*, a circular arc through *c* and *b* will represent the parabola without sensible error. Such an arc must be tangential to *a b* at *b*, and must therefore be drawn from a centre on a line through *b*, perpendicular to *a b*.

The most ordinary case of bridge construction is that in which the structure is itself heavy, and is subject to a travelling load coming on to the bridge from either end. The distribution of shearing force in this case may be conveniently represented by drawing two triangles (Fig. 15), the ordinates of which at the abutments are equal to half the total distributed dead load, and two parabolic (or circular) arcs whose ordinates at the abutments are half the total live load. Then the total vertical breadth of the figure at any point will represent the maximum shearing force at the corresponding point of the span, during the passage of a train from either end.*

FIG. 15.

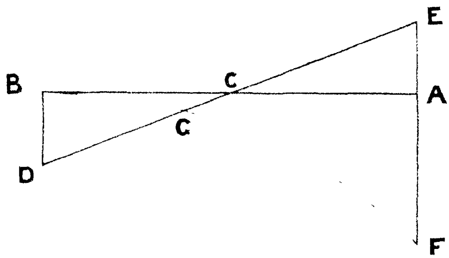


* This construction was originally given by W. B. Blood, Esq.—*Proceedings of the Institution of Civil Engineers*, vol. xi. p. 9.

16. A peculiarity of the stresses due to a travelling load is, that the shearing force over a greater or less portion of the bridge, near the centre, changes sign during the passage of the load.

Let $AB$ (Fig. 16) represent the span of a bridge, then the shearing force due to the dead load, as already shown (§ 12), is represented by the ordinates of two triangles, $AEC$, $CDB$, and counting the forces to the right of the section, is positive from $A$ to $C$, and negative from $C$ to $B$. Suppose a train to come on the bridge from $B$. The successive maxima of

FIG. 16.




the shearing force due to the live load will be represented by the ordinates of a parabola $BFA$, which will be positive on the side of a section towards $A$, but which is so drawn that the breadth of the figure $EFGBD$ represents at any point the algebraic sum of the shearing forces due to dead and live load. In this figure, the shearing force from $A$ to $G$ is positive, so that at any section the part of the girder on the right of the section tends to slide upwards under the action of the shearing force relatively to that on the left. From $G$ to $B$ the shearing force is negative, so that the part of the girder on the right of a section tends to slide downwards relatively to that on the left. It will now be clear, that, over the portion of the bridge from $C$ to $G$, the shearing stress has changed sign during the passage of the live load.

Now a railway bridge is subjected to a dead load and to two live loads, one coming on to the bridge from either end. Three cases therefore arise, represented in Fig. 17, where the first diagram indicates the maximum stresses when the live load arrives on the bridge at the left abutment; the second

FIG. 17.

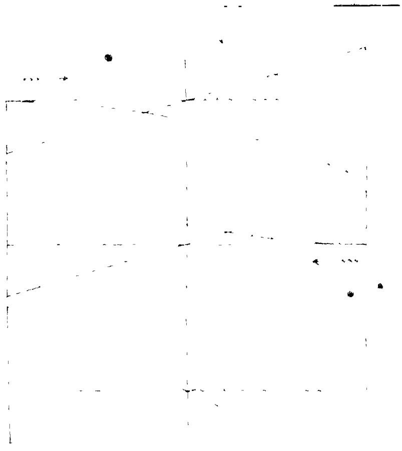
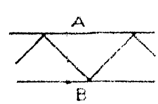


diagram those when the load arrives on the bridge at the right abutment; and the third those due to a uniform live load over the whole of the bridge. The peculiarity brought out by these diagrams is that over a portion of the centre of the girder the shearing force may be + or —, according as the live load arrives on the bridge from the left or the right.

Let Fig. 18 represent part of a braced girder near the centre, and $AB$ a section through a bracing bar. Then the bar cut by the section is a strut or a tie, according as the part of the girder to the right of the section tends to slide up or down, relatively to that on the left; that is, as the shearing force, counted from the right abutment, is positive or negative, according to the convention hitherto employed. In those parts of the girder in which the shearing force may be at different times either + or —, the same bars will be alternately struts and ties. Hence it is that at the centre of braced girders used for railway bridges, the bracing bars require to be so constructed as to stand equally a compressive or tensile strain, and the provision to effect this is called *counterbracing*.

FIG. 18.



17. *Nature of resistance to shearing.* The resistance to the shearing force, whose magnitude has been determined, depends on the mode of distribution of the material.

If the structure is a framed or braced structure, as in Fig. 18, consisting of bars connected by joints, then an extremely simple relation exists between the shearing force and the stress on the bars. In this case the shearing force is resisted by simple tensions and compressions on all the bars cut by the section, and, in order that there may be equilibrium, the sum of the vertical components of the stresses on the bars must be equal and opposite to the shearing force. That is putting $F$ for the shearing force at a section, and $R_1$, $R_2$, for the resistances of the bars, $i_1$, $i_2$, for their inclinations to the horizontal:—

$$F = R_1 \sin i_1 + R_2 \sin i_2 + \ldots$$

and with the shearing force positive on the right of the section the bars will be struts or ties according as they slope downwards to the right or downwards to the left; and conversely if the shearing force to the right of the section is negative. In the case of Fig. 17, the two horizontal bars are not strained by the shearing force, and the stress on the diagonal bar is simply

$$R = F \operatorname{cosec} i = F \frac{l}{d}$$

where $l$ is the length of the bar, and $d$ the depth of the girder.

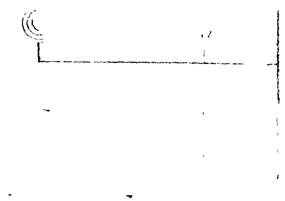
If the beam or girder has a solid or continuous web, then the shearing force gives rise to complex stresses, the nature of which will be more easily explained after the consideration of the distribution of bending moment.

## Graphic Representation of Bending Moment.

18. The bending moment at any section is the moment of the couple formed by the resultant of the external forces on one side of the section, and the equal and opposite resistance to shearing at the section. Its magnitude is the algebraic sum of the products of the forces on one side of the section, and the distances of their directions from the section.

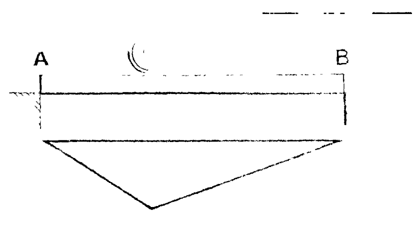
19. ***Beam fixed at one end and loaded at the other.*** In this case there is only the load on the left of any section, and the bending moment which is the product of the load and its distance from the section, is obviously proportional to the distance of the section from the free end of the beam. The bending moment at any section $a$ is therefore the vertical ordinate of a triangle, whose base at the abutment is equal on any scale to $Wl$, where $W$ is the load and $l$ the length of the girder or cantilever.

FIG. 19.



20. *Beam supported at both ends and loaded at intermediate points.* In this case the bending moment is represented by a

FIG. 20.



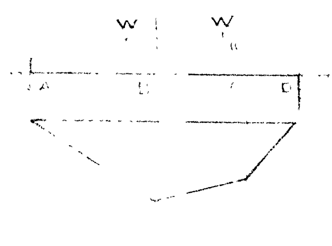

triangle whose apex is vertically under the load, and whose magnitude at that point is

$$M = W\frac{m\,(l - m)}{l}$$

where $W$ is the load, $m$ the distance of the load from either abutment, and $l$ the span.

If the beam is loaded at two points the curve of bending moments is a polygon whose breadth at any point is the sum of the breadths of the two triangles (shown dotted) which represent the bending moments of the

FIG. 21



weights separately. Let the load at $C$ be $W_1$; at $B$, $W_2$. Let $CD = m$; $BD = n$. The reaction at $A$ is

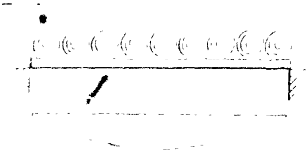
$$P_1 = \frac{m\,W_1 + n\,W_2}{l}.$$

The bending moment at $B = (l - n)\,P_1$. That at $C = (l - m)\,P_1 - (n - m)\,W_2$. Setting off these ordinates the polygon can be drawn.

If in this case the weights are equal and equally distant from the supports, the bending moment between $B$ and $C$ is uniform and equal to $W m$.

21. *Beam uniformly loaded.* Let the load be $w$ per unit of span. The reaction at each abutment is then $\frac{w\,l}{2}$. Let $x$ be the distance of a section from one abutment; the bending moment at that section is $\frac{w\,l}{2}\,x - w\,x\,\frac{x}{2} = \frac{w\,x}{2}\,(l - x)$. This is easily seen to be the equation to a parabola, whose vertex is at the centre of the span.

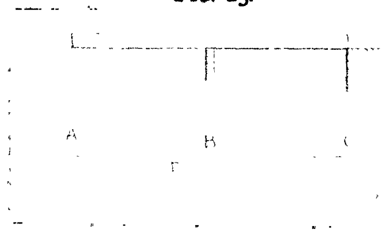
FIG. 22.



When $x = o$ and $x = l$ the bending moment is zero. When $x = \frac{l}{2}$ the bending moment is a maximum and $= \frac{w\,l^2}{8}$. If the scales are so arranged that the versed sine of the arc is not more than one-eighth of the span, a circular arc may be substituted for the parabola.

22. *Uniform beam uniformly loaded on three supports at one level.* As before, if $w$ be the load per unit of span and $l$ the length of one span, the reactions at the supports will be

FIG. 23.



$\frac{3}{8}\,wl$, $\frac{5}{4}\,wl$, and $\frac{3}{8}\,wl$ respectively. Let $x$ be the distance of any section from $A$, then the bending moment at $x$ is $wx\left(\frac{3l}{8}-\frac{x}{2}\right)$. At $A$ and at a point $D$, whose distance from $A$ is equal to $\frac{3l}{4}$, the bending moment vanishes. At $B$ the bending moment is $-\frac{wl^2}{8}$. The curve of bending moments is a parabola through the three points thus given, having its vertex at a horizontal distance $=\frac{3}{8}l$ from $A$, and the curve for the adjoining span is similar.

23. *Curve of maxima of bending moments due to a concentrated travelling load.* When a concentrated load travels over a beam, the distribution of bending moments in each position of the load is represented by a triangle whose apex is under the weight; and the curve through the apexes of all the triangles due to all the positions of the load is a parabola, whose vertical ordinate at the centre of the beam is $Wl \div 4$, and whose ordinates at the abutments are zero.

If a series of concentrated loads, as for instance those due to the wheels of a locomotive, pass over the beam, the curve of maximum bending moments becomes very complex. The following construction will enable such a curve to be drawn. Given the magnitudes of a series of loads $W_1$, $W_2$, passing over a beam and their distances apart, to find the bending moment at any section $C$ due to any position of the loads. At the centre of the span $O$ take ordinates $OF$, $OG$, equal to

FIG. 24.

$W_1 l \div 4$, $W_2 l \div 4$; and through $A$, $B$, and $F$ and $G$ draw the

curves of maximum bending moments $AFB$, $AGB$. Then for any position of the loads the distribution of bending moments is given by the triangles whose apexes are on the corresponding curves vertically under the loads. The bending moment at $C$ due to the assumed position of the loads is the sum of the intercepts $CD$, $CE$, at $C$. The maximum bending moment at $C$ will generally occur with the heaviest of the rolling loads at $C$.

24. *Resistance to bending.* The bending moment at a section is the moment of a couple. In order that there may be equilibrium, a couple of molecular forces of equal moment must be developed at the section by the bending of the beam. Consider a slice of a beam cut off by two planes perpendicular to the longitudinal axis. Under the action of the forces which bend the beam, the edges of this slice are shortened on the concave side and lengthened on the convex side of the bent beam, and the planes originally perpendicular to the straight axis are now perpendicular to the bent axis, or in the direction of radii of curvature; and the two triangular prisms

FIG. 25.

intercepted between the new and the original position of a plane may be called the *strain* prisms of the slice, using the word in the strict sense of alteration of dimension by stress. Now stress is, within the limits at which the material is sensibly elastic, proportional to strain, and hence the stress on any element of area of the section is proportional to the horizontal thickness of the strain prism at that point.

About these two prisms, it should be noted, that one of

them reprsents a shortening of the fibres and consequently a compressive stress, the other a lengthening of the fibres and consequently a tensile stress. If the beam is under the influence of the bending moment alone, this thrust and tension form a couple, of moment equal to the bending moment which produces the deformation. The next point is that the strain, and consequently the stress, at any point is proportional to its distance from the intersection of the bounding planes of the strain prisms, or *neutral axis* of the section. It will not be necessary to investigate, in a general form, the integration of a uniformly varying stress. A simple consideration will show what form a beam should possess, in order, with a given amount of material, to present the greatest resistance to flexure, and what the amount of that resistance is.

It has already been laid down (§ 2) that in a perfect structure the stress should be of uniform intensity. In a solid beam, say of rectangular section, this condition is not fulfilled. Instead of being uniform, the stress is uniformly varying, or in proportion to the distance of any point in a section from the neutral axis. Instead of all the material doing its full duty, only the layers of material at the extreme edges of the section are fully strained, the rest of the material bearing a less and less amount of stress, till at the neutral axis the material, so far as the bending moment is concerned, is perfectly useless, adding to the weight of the structure without adding to its strength. To rearrange the material of the rectangular beam so that it may be uniformly strained, it must be accumulated in two thin plates, flanges, or booms, placed as far apart as possible, and having between them only so much material, in the form of a thin vertical web, as may be necessary to resist the shearing force. We thus get the I-shaped section, now the universal form for girders.

Let $A_1$, $A_2$, be the sectional areas of the top and bottom flanges of such a girder; $h$ the vertical distance between the centres of gravity of the flanges or effective depth of the girder. Then since the molecular forces at the section, in

equilibrium with the bending moment $M$, form a couple whose arm is $h$—,

where $f_1$, $f_2$ are the intensities of the stresses, and $A_1 f_1$, $A_2 f_2$ the total stresses on the top and bottom flanges. In other words, the total stress on either flange is $\frac{M}{h}$, and this divided by the area of the section of the flange, gives the intensity of stress.

The areas of the booms must obviously be inversely proportional to the stresses $f_1$, $f_2$, which experience shows to be safe for tension and compression respectively. The functions of the flanges and web are distinct, the former being proportioned to the bending moment, and the latter to the shearing force. No doubt when the web is a continuous plate it may assist the flanges in resisting flexure to an extent in properly designed wrought-iron girders, of perhaps 5 to 10 per cent. of the whole resistance; and in the same case the booms, which always possess some vertical rigidity, may bear a small fraction of the shearing stress; but, in practice, these quantities are neglected, the booms being assumed to carry the normal, and the web the tangential stress at a section.

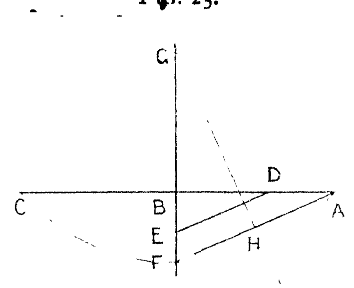
## DISTRIBUTION OF STRESS IN BOOMS.

25. *Girders of uniform depth.* In a girder of uniform depth ($h$ constant) the curves, the ordinates of which represent the total stress in a boom at any point of the span, are precisely similar to those already given for bending moments. In drawing them it is only necessary to substitute for the values of $M$ already given the corresponding values of $M \div h$. Thus, for a uniformly distributed load the curve of stress will be a parabola, whose vertex is at the centre of the span, its ordinates at the abutments zero, and its ordinate at the centre $wl^2 \div 4h$.

For this case, which is that for which a curve of stress is most frequently required, the following construction may

prove useful. Take on any scale $AC =$ to the span, bisect it in $B$, and draw $GF$ at right angles to it. On the same scale take $BD =$ the effective depth of the girder; $BE =$ one-fourth the distributed load on any other scale. Join $DE$; draw $AF$ parallel to $DE$, cutting $GF$ in $F$. Then a parabola through $AFC$, having its vertex at $F$, will be the curve of stress, and the vertical distance between $AC$, and the parabola at any point will be the stress on either flange of the girder at that point, on the same scale as that on which $BE$ is one-fourth the load. If that scale has been so taken that $BF$ is not greater than one-eighth $AC$, bisect $AF$ in $H$, draw $HG$ perpendicular, cutting $GF$ in $G$. Then a circle drawn from centre $G$, with radius $GF$, will sensibly coincide with the parabolic curve of stress.

FIG. 25.




## SHEARING STRESS IN A CONTINUOUS WEB.

26. One question of pure theory remains for consideration, namely—the mode in which a plate or continuous web resists the shearing force. Probably no part of the theory of beams is so imperfectly understood by practical Engineers, or so puzzled the first constructors of plate web bridges. Perhaps it is only from the fortunate accident that the plate web must, in general, from quite other considerations than the amount of its resistance, contain an excess of strength, which has prevented disaster in the many cases in which, whilst the greatest care has been expended on the design of the flanges of a girder, the web has been proportioned by mere guess and rule of thumb.

The subject of stress in a continuous web has been treated with great thoroughness by Professor Rankine, and the following remarks dealing with the general conceptions, which

should be in the mind of the Engineer rather than with the strict mathematical investigation, may be regarded as the illustration of principles which he has already very clearly developed.

In treatises on the theory of beams it is sometimes broadly asserted that we know nothing of the direction of the stresses in a continuous web. If this were true the web could not be scientifically designed at all. It is indeed true that there are no defined channels along which, and along which alone, stress is transmitted. Every line in the beam, unless on the neutral surface, is a line along which the material is either extended or compressed, and which, therefore, whether straight or curved, represents a channel along which force is transmitted. The possibility of knowing definitely the state of strain at any point of the web depends on our being able to resolve all the forces, acting on a material particle at that point, into simple tensions and compressions; and finally to combine these until there is one simple tension and one simple compression, representing and equivalent to all the other forces. This tension and compression are called the principal stresses, and the lines which mark their directions are called the lines of principal stress. By laying down the lines of principal stress for a number of points we may form a tolerably clear conception of the condition of stress in a plate web.

The simplest conception we can form of shearing stress, and that from which it derives its name, is that it is the resistance at a vertical section to the force tending to cause one side of the section to slide relatively to the other. (§ 8). Under the action of the loads the beam tends to give way in the manner roughly indicated in Fig. 26. Suppose, however the beam actually cut into horizontal plates or laminæ. Under the bending action each lamina would slide on the one below it, in the manner shown in Fig. 27, which is precisely analogous to sliding in vertical

FIG. 26.

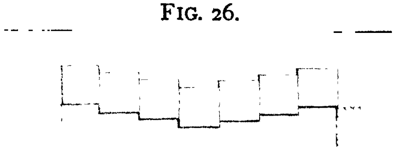
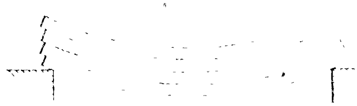


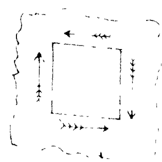
FIG. 27.



planes. Now in an actual solid beam this sliding cannot take place, and there must therefore be generated, on horizontal planes, stresses due to the resistance to sliding, equivalent to the forces which would have to be applied to the laminæ in Fig. 27, to bring them to the position they would occupy if the beam were solid.

Suppose a small square engraved on the web of a beam near the neutral axis, where the material is subject to shearing stress only, then let the beam be bent, and the square will be found to have been distorted into a rhomb. Consider what forces must be applied to the faces of the prism, of which the square is the base, to cause this distortion. A little consideration would show that a pair of equal and opposite forces must be applied to each pair of faces, each pair of forces forming a couple, and equilibrium requiring that the moments of the two couples should be equal and opposite.

FIG. 28.

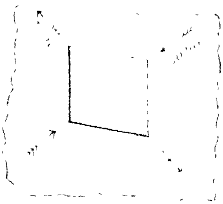


Since, by the assumption that the prism is rhombic, the faces are all equal, and the opposite faces at equal distances, the intensity of the tangential stress on each face must be equal. So that for any pure shearing stress the intensity of the stress must be equal on two planes initially perpendicular to each other. The stress on the vertical faces of the prism corresponds with the tendency to sliding at vertical planes (Fig. 26), and that on the horizontal planes to the tendency to sliding on horizontal planes (Fig. 27). It is now shown that those tendencies are equal.

Hence a shear upon a given plane in a continuous plate cannot exist alone, but must be combined with a shear of equal intensity on a different plane. The four tangential stresses (Fig. 28) have the effect of lengthening one diagonal and shortening the other, and the same deformation may be conceived to be produced by a simple thrust and tension, acting at right angles to each other along the diagonals of

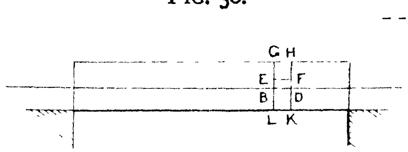
the rhomb. And by resolving the tangential stresses in the direction of the diagonals it may be shown that the intensity of the tension and compression, on planes through the diagonals of the rhomb is equal to the intensity of the tangential stress on either of the faces. Thus in the ultimate analysis, the shearing force on any small portion of the beam may be resolved into a single pair of simple stresses at right angles to each other.

FIG. 29.



27. Consider a thin prism $HFEG$, bounded by two vertical sections of a beam at a distance $\Delta x$ from each other. On the face $HF$ there will be a thrust $T$, and on the face $GE$ a thrust $T + \Delta T$. The difference or the thrust $\Delta T$ must be

FIG. 30.




resisted by a tangential stress on the face $FE$. The value of $T$ and $T + \Delta T$ will increase from zero at the edge of the beam $HG$ to a maximum at the neutral axis $DB$. Let $M$ be the bending moment at $HK$. Then the thrust on $HD$ is $\frac{M}{h}$ where $h$ is the effective depth of the girder,* and that on $GB$ is $\frac{M + \Delta M}{h}$. Hence the tangential stress on $DB$ is $\frac{\Delta M}{h}$ Let $t$ be the thickness of the beam at $D$. Then the intensity of the stress on $DB$ is $\frac{\Delta M}{ht \,.\, \Delta x}$, or in the limit $\frac{dM}{ht \,.\, dx}$. But

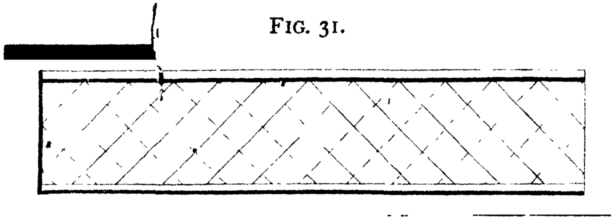
* The term *effective depth* is here used for the distance between the resultants of the thrusts above and tensions below $BD$. In the case of a flanged beam, and it is these only with which the builder of bridges is concerned, the effective depth is sensibly equal to the distance from centre to centre of flanges.

$\frac{dM}{dx} = F$, the shearing force at $HK$.* Hence the intensity of the tangential shearing force on a horizontal, and consequently also on a vertical plane, at $D$, is $\frac{F}{ht}$.

In a beam in which the great mass of the material is collected in the flanges, and nearly the whole thrust is borne by the top flange, the value of $\Delta T$, and consequently the intensity of the shearing stress on a web of uniform thickness will increase only by a small amount, between the point of junction of the web and boom, and the neutral axis. In the same way, nearly the whole amount of tension being borne by the bottom flange, the intensity of the shearing stress will increase very slightly between the bottom flange and the neutral axis. In other words, for the whole depth of the web the intensity of the shearing force on horizontal or vertical sections will be very nearly constant and equal to $\frac{F}{ht}$. This uniformity of shearing stress, joined with the fact that nearly all the horizontal stresses are carried by the flanges, has this important consequence:—If on any part of the web of a flanged girder we take a prism like that in Fig 28, with horizontal and vertical faces, the intensity of the shearing stress on the faces will be $\frac{F}{ht}$; and the simple tension and compression in the direction of the diagonals of the base of the prism will have the same intensity, and will be inclined at angles of 45° with the neutral axis. Hence, approximately, over the whole surface of the web, the lines of principal stress will be at angles of 45° with the neutral axis. Representing tensions by dotted, and compressions by full lines, the whole state of stress in the web of a beam will be represented by a network of lines at right angles to each other, and inclined at 45° with the axis of the beam. (Fig. 31.) In beams supported at both ends

* That $\frac{dM}{dx} = F$ is easily seen by differentiating the expression for $M$ in § 8 with respect to $x$. The result is the expression for $F$ in the same article.

FIG. 31.



these lines vanish, and change sign at the section of the beam, for which the shearing force vanishes; or in symmetrically loaded beams at the centre.

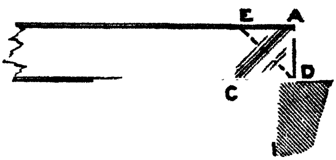
28. In proportioning the web of a beam, if of uniform section throughout, we should only require to provide material along lines measured at an angle of 45° with the neutral axis, to resist a tension and compression of $\frac{F}{ht}$ tons per square inch. ($F$ being in tons, and $h$ and $t$ in inches.) In actual practice, however, plate webs have generally horizontal and vertical joints, which, as regards tensile and shearing strains, are lines of weakness. There must, therefore, be provided in the line of fracture of the vertical and horizontal joints, material sufficient to resist a shearing force of $\frac{F}{ht}$ tons per square inch; and along lines inclined at 45° with the axis, material to resist a thrust of the same intensity. In most cases the web will be weakest in resisting flexure from the thrust, and it is to stiffen the web against this tendency that the heavy vertical pillars of T iron are rivetted over the joints of the web.

29. It may be interesting to point out some cases in which the webs of beams have given way before the flanges. In that storehouse of information on the strength of beams, the experiments preliminary to the construction of the Conway and Menai tubular bridges, instances of this kind may be found.* In most of the rectangular tubular beams which were subjected to experiment, the resistance of the sides was excessive, and in most instances the beam gave way by the

* *The Britannia and Conway Tubular Bridges*, Edwin Clark, vol. I, pp. 165, 410, 414, 439.

buckling of the top flange. But *in the sixteenth experiment,* on a tube with $\frac{3}{8}$-in. top plate, $\frac{1}{4}$-in. bottom plate, and $\frac{1}{8}$-in. sides, the weakness of the last to resist the diagonal compressive strains began to manifest itself. The sides buckled *very much at the ends*, where the shearing force was greatest, but the tube ultimately gave way at a joint in the bottom plate. *The tube was then straightened and repaired, and* loaded at a point 6 ft. from one support and 24 *ft. from the* other. Then, with as small a load as 7,300 lbs., there was developed a buckle or corrugation of the sides, at the end where the shearing force was greatest, the ridge of which was along the line *AC*. Comparing this with the previous figure, it will be seen that the plane of flexure, *DE*, corresponds with the direction of the diagonal thrust at that point. In the thirtieth experiment a tube failed from distortion of the sides near the ends in a similar manner. In the fiftieth experiment, made by Mr. Brunel, on a large girder of 66 ft. span and 10 ft. depth, failure also took place through the giving way of the web at one end of the girder. The web began to buckle with 110 tons, and the buckle increased until, with 165 tons, the sides collapsed under the compression at the ends, *shearing* the rivets in the vertical joints of the web nearest *the ends* of the girder, the booms being uninjured. The same girder, with the sides strengthened, bore 188 tons before giving way fairly in the booms. The danger of failure from weakness of the web is therefore no imaginary one.

FIG. 32.




In the first two experiments on the model of the Britannia bridge, in which the sides were simple plates without stiffening except at the ends, the existence of compressive strains in the sides was very clearly indicated; and in the second experiment the weakness of the sides to resist thrust led to the failure of the girder. *The sides were thrown by the stress into diagonal waves of puckering, and it would have been* very interesting if an exact diagram of these had been preserved. It was in consequence of this experiment that the

T iron stiffening pillars were applied to the web, which have since been universally employed in girders with plate webs.

(A beautiful method of drawing polygons of bending moment, or of total stress, and at the same time determining graphically the supporting forces, has been described by Mr. J. H. COTTERILL, M.A., in "ENGINEERING" for January 8th, 1869.)

# LECTURE II.

## *ON THE INTENSITY OF STRESS IN BRIDGES, AND ON ESTIMATING THE LOAD.*

In the last Lecture, attention was directed to the laws of the distribution of stress in transversely-loaded structures. The first step to be taken in the design of a girder is to determine the limiting intensity of stress, consistent with the safety and durability of the structure.

### LIMITS OF INTENSITY OF STRESS.

30. *Attempts at the experimental determination of the limits of stress consistent with safety.* Many years ago, Dr. Fairbairn and Professor Hodgkinson made the earliest attempt to ascertain, by direct experiment, the extent to which cast-iron bars could be permanently loaded without injury. There had been a notion amongst the earlier experimenters on the strength of iron that there was a defined limit of stress at which iron began to take a permanent set, which was called the elastic limit. With stresses less than that corresponding to the elastic limit, the elongation or strain was supposed to be rigidly proportional to the stress; with greater stresses the elongation was known to be no longer proportional to the stress, and the iron after loading was found to have been permanently distorted. This elastic limit was conveniently supposed to mark the extent to which the material could be loaded without any permanent alteration of structure, and therefore with safety. Very careful and accurate experiments of Professor Hodgkinson showed that when loaded for the

first time, even small weights produced a permanent set in long bars, perceptible when the means of measurement were sufficiently delicate. But they also showed that when a bar had once taken a set under a given load, a repetition of the same amount of load produced no perceptible increase of set, unless the load was a large fraction of the breaking weight. The idea that the point at which a bar first took a measurable set was the elastic limit was thus shown to be fallacious, but at the same time there was suggested a new test. If up to a certain limit of stress the elongation of a bar was always the same, with any number of repetitions of the load producing it, and beyond that limit the elongation increased with repetitions of the load, that limit would mark the extent to which the loading could be carried without causing a progressive deterioration of the bar; and whatever rule might be found to apply to a bar under a tensile or transverse stress might be assumed to apply also to other descriptions of strain, on which experiments are less easily made.

Dr. Fairbairn therefore determined to try and ascertain with what loads the deflection of cast-iron bars, transversely loaded, would gradually increase in course of time. For it is evident that if with a given load the deflection of a bar gradually increases, the bar will ultimately break down with the load. In the first experiments, cast-iron bars were loaded and left in repose, in the same condition as a structure strained by a dead load. In this condition the deflection of a bar loaded with 5-8ths of the breaking-weight increased in five years from 1·27 to 1·31 inches. The deflection of a bar loaded with 3-4ths of the breaking weight increased from 1·46 to 1·62 inches. With all the bars on which the load amounted to 5-8ths of the breaking-weight the deflection progressively increased in course of time. It appears therefore that cast-iron bars could not be permanently safe if loaded with 5-8ths of the breaking-weight.

Making due allowance for the contingencies of practice, these experiments, so far as they go, confirm the rule that the permanent stresses on structures should not exceed 1-3rd of those at which small bars would break. Thus, if the ulti-

mate strength of the iron of a bridge, when tested, is 21 tons per square inch, 7 tons may be assumed to be the maximum stress which should be permitted in a structure subject to a dead load only.

31. The case of a live load is different. We know, theoretically, that a load suddenly imposed on an elastic bar produces, momentarily, twice the deflection and twice the stress, which would be produced by the same load resting quietly on the bar. A railway train at high velocity is a load which is rapidly, but not quite suddenly imposed, and will cause stresses greater than those due to a quiescent load. In addition to this the vibrations to which it gives rise produce local augmentations of the stress in a degree not susceptible of calculation. In the experiments of the Royal Commission on Railway Structures, in 1849, when a truck weighing 1,120 lbs. was run over a pair of light cast-iron bars at a velocity of thirty miles an hour, the maximum deflection was twice as great as when the same load quietly rested on the bars. With 2,066 lbs. the dynamical deflection was three times as great as the statical deflection. But the theory deduced from the experiments led Professor Willis to the conclusion that effects of this kind, produced by a rolling load, and alarmingly manifested in experiments on a small scale, would be so greatly diminished with bridges of large dimensions as to be of comparatively little importance. In experiments on the Ewell and Godstone bridges, the former of 48, the latter of 30 feet span, the maximum deflection obtained by running an engine and tender over the bridge was greater than the statical deflection of the same load by 1-7th in the former case, and 1-3rd in the latter.

In experiments on bars loaded and unloaded alternately, they found that if the deflection did not exceed that due to a statical load amounting to 1-3rd of the breaking-weight, the bars suffered 10,000 repetitions of load without appreciable injury; but one bar broke after 50,000 repetitions of load; and when the deflection was increased so as to be equivalent to that produced by a dead load of one-half the breaking-weight, all the bars broke after a greater or less length of time.

A few years ago, the question of the safe limits of stress in wrought-iron bridges was investigated in a very interesting and complete experiment by Dr. Fairbairn. A wrought-iron rivetted girder was so placed that a load could be rapidly but not suddenly lowered upon it, and raised again any number of times. Thus arranged, the beam was loaded with 1-4th the known statical breaking-weight, and suffered half-a-million repetitions of loading without appreciable injury. The load was then increased to 2-7ths, and afterwards to 2-5ths of the statical breaking-weight, when the beam gave way almost immediately. It was then repaired, and suffered three million repetitions of loading with 1-4th of the breaking-weight, and ultimately broke a second time after 31,300 repetitions of a load of 1-3rd the breaking-weight.

If we calculate from the loads the stresses which they would produce on the flanges, if quiescent on the girder, we find that the beam gave way with loads producing (as dead loads) a tensile stress, on the net section of the metal, of 7·3 and 6·25 tons per square inch. It is obvious, therefore, that live loads of that magnitude would not have been safe, and that if the stresses due to the live load be calculated in the same way as for dead loads, a much lower limit of stress must be adopted. If 7 tons per square inch be the limiting stress in tension for a dead load, 3½ tons per square inch does not appear to allow too large a margin for a live load.

As to the safe limits of stress in compression no experimental evidence has been obtained, and it is, therefore, assumed that the safe stress, like the ultimate strength, is equal in tension and compression when flexure is absolutely prevented; and about 1-5th less in compression than in tension in ordinary cases, in which though not absent, the amount of flexure is comparatively small.

32. Thus both theory and experiment agree with practical experience in indicating that a rolling load, like a railway train, produces much more injurious strains than an equal quiescent load; and for practical purposes, a rolling load may be assumed to be equivalent to a dead load of twice its magnitude. In calculating the sectional areas of metal for a

bridge, therefore, since the precise stresses to which a live load will give rise cannot be ascertained, either of two courses may be pursued. (1) The load may be reduced to an equivalent dead load, whose magnitude will be twice the actual live load + the dead load. In proportioning the girder a limiting stress suitable to a dead load may then be taken. (2) The stresses may be calculated from the actual load, considered as a dead load, and a variable limiting stress may be adopted, dependent on the ratio of the dead to the live load. The following Table gives such values for the limiting stress as would be equivalent to allowing twice as much metal for a given live load, as for an equal dead load.

*Safe Limits of Stress.*

| Ratio of Live load to Dead load. | Tons per square inch. Tension. | Compression. |
|---|---|---|
| All Dead load. | 7·00 | 5·50 |
| ·25 | 5·83 | 4·59 |
| ·33 | 5·60 | 4·40 |
| .50 | 5·22 | 4·12 |
| ·66 | 5·00 | 3·93 |
| 1·00 | 4·66 | 3·66 |
| 2·00 | 4·20 | 3·30 |
| All Live load | 3·50 | 2.75 |

Some of the stresses given as safe in this Table may appear large; but the Britannia bridge is strained to the extent of 7 tons per square inch in tension on the passage of a train, yet it gives no indications of weakness—a result due doubtless to the fact that in that bridge the live load is only about one-fourth as great as the dead load. On the other hand, Dr. Fairbairn's experiment shows that a stress of that magnitude would certainly be unsafe on a small girder, the weight of which formed an insignificant part of the whole load.

33. *Limits of stress adopted in practice.* Although the method of proportioning bridges just indicated is the really scientific method, it has not hitherto been often followed in practice. The plan adopted has been to calculate the stresses

due to the total load as if quiescent, and to proportion all bridges to a stress lying between the limits which have just been assigned for dead and live loads respectively. The most usual maximum stress permitted in practice is 5 tons per square inch of net section of metal in tension, and 4 tons per square inch of gross section in compression. The French engineers, very conveniently, calculate the stress on the gross section without deduction for rivet holes, both in tension and compression, and adopt the same limiting stress in both cases, namely—6 to 7 kilogrammes per square millimetre, or 3·8 to 4·5 tons per square inch.

The following Table gives the maximum intensity of stress in a few well-known bridges of recent construction, the stress in tension being calculated on the net section of the metal exclusive of rivet holes, and that in compression on the gross section.

*Limits of Stress in Bridges.*

| | Tons per square inch. | |
|---|---|---|
| | Tension. | Compression. |
| Cannon Street, Plate Girder, Charing Cross, Lattice ... Connecticut, N. Truss ... | 5 | 4 |
| Passau, Lattice ... ... ... | $5\frac{1}{2}$ to 6 | $4\frac{1}{4}$ to $5\frac{1}{4}$ |
| Penrith, Tubular Girder ... | $4\frac{3}{4}$ | $4\frac{1}{4}$ |
| Place de l'Europe, Lattice ... | $4\frac{1}{4}$ | $3\frac{3}{4}$ |
| Lough Ken, Bowstring ... | 4 | $3\frac{3}{8}$ |
| Isère, Lattice ... ... ... | $4\frac{1}{2}$ | $3\frac{1}{2}$ |

34. *Limit of thrust on a strut.* When a member of a girder, of which the length is considerable in proportion to the diameter, is subject to a thrust, it is not only compressed, but bent. The effect is that, at certain points, the maximum intensity of stress is greater than the average intensity. If a defined limit of stress is adopted, then the thrust which may be transmitted is less than that which would be safe if there were no bending. Let $T$ be the total thrust in tons per square inch on a strut, $t$ the average intensity of the thrust, so that if $a$ be the sectional area of the strut—

$$T = at$$

Let $l$ be the length, $h$ the diameter of the strut in the direction in which it tends to bend, $f$ the limit of stress in compression, which is adopted for the structure, in tons per square inch. Then, if the strut were not subject to bending, we should have

$$t = f,$$

but if the strut is long in proportion to its diameter, and is fixed at the ends, the safe intensity of thrust is*—

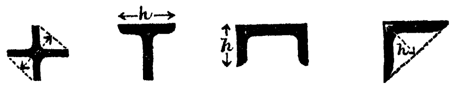
$$t = f \div \left(1 + \frac{l^2}{3000h^2}\right)$$

and if the strut is unfixed at the ends—

$$t = f \div \left(1 + \frac{l^2}{750h^2}\right)$$

In applying these formulæ to the struts of bridges, the strut is to be considered unfixed as regards flexure in the plane of the girder, if attached to the boom by a bolt; fixed if attached by a series of rivets. If the girder is a Warren girder, or the struts and ties are unconnected at their intersection, the strut is equally free to bend in any direction, and it will bend in the direction in which $h$ is least, as in the following figures. In lattice girders, in which the struts and

FIG. 34.




ties intersect and are rivetted together at the intersections, it is generally sufficient to consider the tendency to flexure in the plane of the girder, and at right angles to that plane. If, as is almost always the case, the ties are flat bars, with the greatest width in the plane of the girder, they powerfully resist the tendency of the strut to bend in that plane, but render it very little assistance in a direction at right angles to that plane. To compensate this in girders with a double web, two parallel struts are braced together, to form a single strut. Hence, in calculating the limit of thrust for such struts, we may take for flexure in the plane of the girder, $l$ equal to the distance between the intersection of the struts

* *See* Note, page 49.

and ties only; and for flexure at right angles to that plane $l$, equal to the whole length of the strut, but $h$ the width across the two struts rivetted together.

## ESTIMATION OF THE LOAD ON BRIDGES.

35. *Ratio of the weight of a girder to its load.* I culations with respect to bridges, it is constantly necessary to form an approximate estimate of the weight of a beam, the load on which, exclusive of its own weight, is already known. Various methods of proceeding have been proposed to compass this end, but no method appears more convenient than to express the weight of a beam as a function of its load. To do this with perfect accuracy involves formulæ of great complexity, but looking at the fact that from 2-3rds to 3-4ths of the whole weight of metal in a girder is concentrated in the top and bottom booms, and that the volume of a well-designed boom should be proportional to its length, and to the area of its section at the centre, it appears probable than an approximate formula might be found to express the weight in terms of the load and the stress at the centre. The following formula is of this kind, and gives results of considerable accuracy, when used as an interpolation formula, to obtain the weight of a girder of given dimensions from that of a similar girder of different dimensions.

Let $W$ = the total external distributed load in tons (exclusive of weight of girder).

$W'$ = weight of girder itself in tons.

$l$ = clear span in feet.

$d$ = effective depth in feet.

$s$ = average stress in tons per square inch on the gross section of the booms, at the centre.*

$r$ = ratio of span to depth.

$A$ = gross area of two booms at centre in square inches.

* The most usual value for $s$ is 4. If $s_1$ = stress per square inch on gross section in compression, and $s_2$ = stress per square inch of net section in tension, we may take

$$s = \frac{1}{2}\left(s_1 + \frac{5}{6}s_2\right)$$

($W$ and $W'$ are the weights of length corresponding to clear span.)

Then—

$$W' = \frac{Wl^2}{Cds - l^2} = \frac{Wlr}{Cs - lr} \quad . \quad . \quad . \quad . \quad (1)$$

And to deduce $C$ from girders of known weight—

$$C = \frac{(W + W')\, l^2}{W'ds} = \frac{4\, Al}{W'}$$

The values of $C$ for the main girders of large bridges will be given presently. For small wrought-iron plate girders $C$ may be taken at from 1,400 to 1,500.

36. *Estimate of live load on bridge.* The weight of a crowd of people has been taken for suspension bridges at 40 to 45 lbs. per square foot of roadway. The actual weight of a body of troops on the march is 35 lbs. per square foot, but a standing crowd, if densely packed, may reach a weight of 84 lbs. to the square foot. The tendency now is to allow in all structures a somewhat larger margin of safety than used to be considered necessary, and hence the load on the footways of girder bridges is generally taken at 70 lbs. per square foot.

For bridges carrying road traffic, from 80 to 120 lbs. per square foot of roadway may be allowed, the latter being the load assumed in designing Westminster bridge. It should not be forgotten, however, that road bridges have frequently to carry very heavy *concentrated* loads. In a recent discussion it was shown that loads of 34 tons, on four or six wheels, sometimes pass over metropolitan bridges.

For railway bridges, it was the practice some years ago to take the rolling load at one ton per foot run for a single line of way, and $1\frac{1}{2}$ tons for a double line; but it is rarely taken now at less than one ton for each line of way, and sometimes, if the traffic is heavy, $1\frac{1}{4}$ tons are allowed to each line of ordinary narrow gauge railway. On small span bridges, a larger allowance must be made. Locomotive engines frequently weigh over 30 tons, on a wheel base of 15 feet, the weight in that case being much greater than the average weight of a heavy train, which scarcely ever exceeds a ton to the foot.

37. *Live load on longitudinal girders.* It is also in small bridges, and especially in the longitudinal girders which form part of the platform of larger bridges, that the assumption that the load is uniformly distributed becomes seriously erroneous. The load on a pair of engine wheels varies from 9 to 14, and in rare instances to 16 tons. And that load, in place of being distributed over 9 to 16 feet of the length of the bridge, is concentrated at a single point, and will, on small spans, produce bending moments of a different character from those due to a uniform load. It is necessary, therefore, to investigate in such cases the effect of the loading much more strictly. Locomotive engines, in working order, reach in English practice a weight of from 30 to 36 tons, and may be taken to have a minimum wheel base of 15 feet, and a minimum length of 25 feet. Heavy tank engines are used on a few lines, reaching a weight of 45 tons, on a 15-feet wheel base, and of a total length of 30 feet. On the Great Northern of France, huge 12-coupled engines have been constructed of 59 tons weight, on a wheel base of 20 feet, and of a total length of 37 feet. These data give for the maximum weight of locomotives $2\frac{1}{2}$ to 3 tons per foot of wheel base, and $1\frac{1}{2}$ tons per foot of total length.

On short bridges, therefore, the average load is excessive, and the strains are greater still from the concentration of the loads at points whose distance is large relatively to the span. Let $P$ be the assumed maximum load on one pair of locomotive wheels, in tons; $d$ the average distance apart of the axles; $l$ the span of the girders; suppose a train to come on to the girders at the left abutment, and let it be required to find the maximum bending moment at a section whose distance from the right abutment is $x$; let $m$ be the next lower integer to $\frac{x}{d}$ and $n$ that to $\frac{l-x}{d}$. Then it is not difficult to show that the maximum shearing stress at a section between the centre and right abutment, at a distance $x$ from the latter, will arise when the position of the train is such that there is one pair of wheels at $x$ and $n$ pairs between $x$ and the left abutment. The maximum bending moment will

arise when there is a pair of wheels at $x$, with $m$ pairs to the right and $n$ pairs to the left, on the girder.

In the former case, the shearing force to the right of the section on the girders carrying one line of way is—

$$\frac{P\,(n+1)}{l}\left(l - x - \frac{nd}{2}\right)$$

At the abutment the shearing force is—

| | | | |
|---|---|---|---|
| For spans less than | $d$ | ... ... | $P$ |
| ,, ,, | $2d$ | ... ... | $P\,(2l - d) \div l$ |
| ,, ,, | $3d$ | ... ... | $3P\,(l - d) \div l$ |
| ,, ,, | $4d$ | ... ... | $2P\,(2l - 3d) \div l$ |

With one pair of wheels at the section, the maximum bending moment on the girders carrying one line of way is—

$$P\left\{\frac{(m+n+1)\,x}{l}\left(l - x + \frac{(m-n)\,d}{2}\right) - \frac{dm}{2}(m+1)\right\}$$

For the bending moment at the centre of the span—

$$(m = n \text{ and } x = \tfrac{1}{2}\,l)\,;$$

$$\tfrac{1}{2}\,P\left\{\frac{2m+1}{2}\,l - (m+1)\,md\right\}$$

which gives the following results—

| | | | Bending moment at centre. |
|---|---|---|---|
| For spans less than | $2d$ | ... ... ... | $\frac{1}{4}\,Pl$ |
| ,, ,, | $4d$ | ... ... ... | $\frac{1}{4}\,P\,(3l - 4d)$ |
| ,, ,, | $6d$ | ... ... ... | $\frac{1}{4}\,P\,(5l - 12d)$ |

The bending moment at the centre of a girder due to an uniformly distributed load of $w$ tons per unit of span is $\frac{wl^2}{8}$; by equating this to the bending moments given above, the following values are obtained for the intensity of a uniformly distributed load, which would produce the same stress at the centre section of the girders as the actual concentrated loads of the locomotive wheels:—

| Span in feet $l$. | | Equivalent distributed load, $w$ in tons per foot. |
|---|---|---|
| Less than $2d$ | ... ... ... ... | $2P \div l$ |
| ,, ,, $4d$ | ... ... ... ... | $2P\,(3l - 4d) \div l^2$ |
| ,, ,, $6d$ | ... ... ... ... | $2P\,(5l - 12d) \div l^2$ |

These formulæ are easily applied, and the weights so ascertained are more convenient in designing the centre section of the girders than the actual loads. As an example of their application, let $P = 15$ tons, $d = 7\frac{1}{2}$ feet. Then the equivalent distributed load per foot run over the girders carrying one line of way is—

| $l$ | $w$ |
|---|---|
| 5 | 6 |
| $7\frac{1}{2}$ | 4 |
| 10 | 3 |
| $12\frac{1}{2}$ | 2·4 |
| 15 | 2·0 |
| $17\frac{1}{2}$ | 2·2 |
| 20 | 2·2 |
| 25 | 2·2 |

In Plate I. the curves of maximum stress for spans of 10, 15, and 20 feet, due to the passage over these bridges of three selected typical engines of the heaviest class, have been carefully drawn. The load on each pair of wheels, and distance of axles assumed in drawing these curves, were—

*Express Engine.*

| | Leading. | | Driving. | | Trailing. |
|---|---|---|---|---|---|
| Loads... ... ... | 9 | | 15 | | 7 |
| Distances ... ... | | 8 | | 8 | |

*Six-Coupled Goods Engine.*

| | | | | | |
|---|---|---|---|---|---|
| Loads... ... ... | 11 | | 12 | | 10 |
| Distances ... ... | | $7\frac{1}{2}$ | | $7\frac{1}{2}$ | |

*Four-Coupled Tank Engine.*

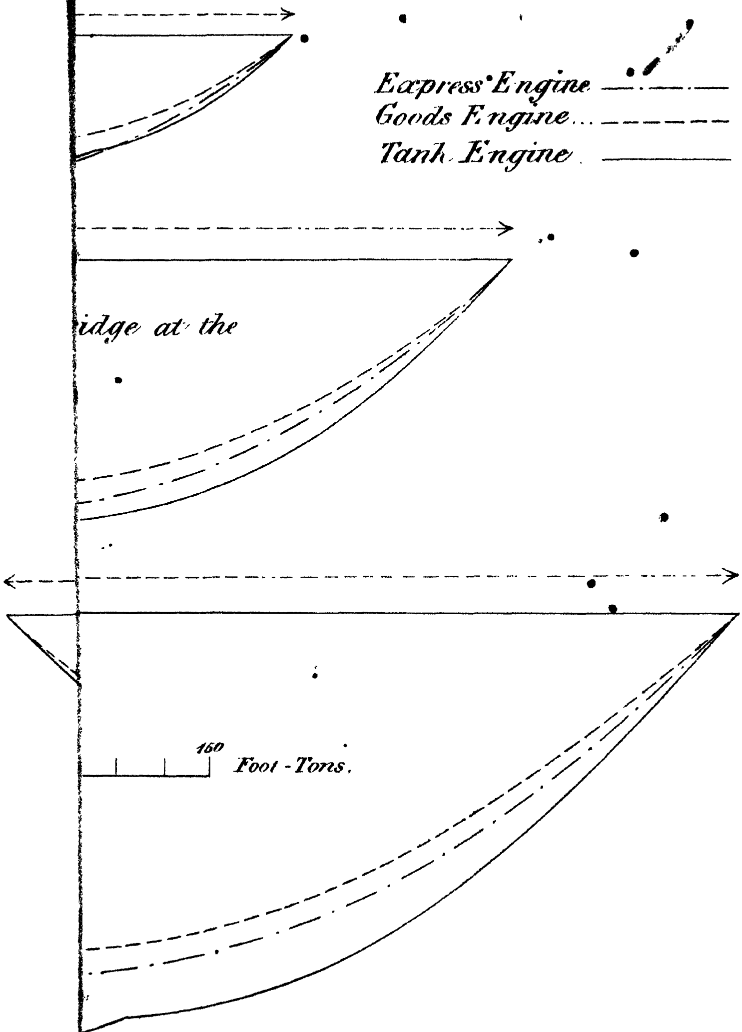
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Loads... ... ... | $7\frac{1}{2}$ | | $7\frac{1}{2}$ | | $14\frac{1}{2}$ | | $14\frac{1}{2}$ |
| Distances ... ... | | $5\frac{2}{3}$ | | 7 | | 8 | |

The bending moments represented in the plate are those due to the whole weight of the engine, and are therefore to be considered as distributed over all the girders carrying one line of way. By scaling off the moment at any section, it will easily be seen how much it is in excess of that due to a

Plate I.

*Moment.*

*Engine.*



distributed load equal to the average weight of a train, and that, on the other hand, it may be fairly represented by that due to the "equivalent distributed load," calculated as above.

Excessive loads of this kind will not come on bridges of more than about 20 feet, because the low weight of the tender reduces the average load per foot in tender engines, and the great length over the buffers reduces it in tank engines. But it would be a wise precaution to take the rolling load for spans of 25 feet at 2 tons per foot; for spans of 30 feet at $1\frac{3}{4}$ tons per foot; for spans of 40 feet at $1\frac{1}{2}$ tons per foot; and for spans of 60 feet at $1\frac{1}{4}$ tons per foot, for each line of way.

38. *Live load on cross beams.* The cross beams of a railway bridge platform either receive the load directly from the timber sleepers or from the longitudinal girders under each rail. In the former case they should be considered as carrying the maximum load on a single pair of wheels, or 15 tons for each line of way. In the latter case, suppose as before, that the effect of a locomotive engine is represented by 15 ton loads at $7\frac{1}{2}$ feet distances. Then, when the cross beams are spaced at distances not greater than $7\frac{1}{2}$ feet, the maximum rolling load is 15 tons for each line of way. For greater distances the total load may be taken at $2\frac{1}{4}$ tons per foot of distance apart, for each line of way. These loads are not equally distributed, but the polygon of bending moments may easily be drawn, and will be found to approximate to the curve due to a distributed load of the following amounts, $l$ being the distance apart of cross beams.

(1) Single line between main girders; span of cross beams 14 feet; equivalent total distributed load on cross beams $= 2\frac{3}{4}\,l$ tons.

(2) Double line between two main girders; span 26 feet; equivalent load $5\frac{1}{4}\,l$ tons.

(3) Double line on top of two main girders 18 feet apart; span 18 feet; equivalent load $= 3l$ tons.

39. *Dead load of permanent way and platform.* The following tables give the average weight of the platforms of

road and railway bridges, with sufficient accuracy for the general calculations on the strength of the main girders:—

*Road Bridges.*

(1) Timber platform on cross girders.

| | | |
|---|---|---|
| Timber and ballast... | 90 lbs. per square foot. |
| Cross beams ... ... | 15 to 20 | ,, ,, |

(2) Brick arches on cross beams.

| | | |
|---|---|---|
| Brick arches ... .. ... | 48 | lbs. per square foot. |
| Concrete ... ... ... ... | 36 | ,, ,, |
| Asphalte ... ... ... ... | 6 | ,, ,, |
| Metalling ... ... ... ... | 118 | ,, ,, |
| Cross beams ... ... ... | 11 | ,, ,, |

*Railway Bridges.*

| | | |
|---|---|---|
| Rails ... ... | ·03 | tons per foot run of each line |
| Ballast ... ... | ·15 to ·21 | of way. |
| Timber ... ... | ·07 to ·17 | |
| | ·25 to ·41 | |
| Platform girders | ·10 to ·25 | |

The weight of the iron cross beams and longitudinal girders under rails may be estimated by the formulæ in § 35, for the live loads already given. Thus, suppose the weight of the platform girders for a single line bridge required, the cross beams being 12 feet apart. The live load on the longitudinals 12 feet long will be (§ 37) $2\frac{1}{4} \times 12 = 15$ tons. Adding to this $0·3 \times 12 = 3·6$ tons for rails, timber, and ballast, we get $W = 18·6$; let $s = 4$, $r = 12$, $C = 1{,}500$. Then the weight of longitudinals—

$$W' = \frac{18·6 \times 12 \times 12}{1500 \times 4 - 12 \times 12} = ·45 \text{ tons;}$$

or, ·037 tons per foot run. The live load on cross beams (§ 38) $= 2\frac{3}{4} \times 12 = 32$ tons; dead load of rails, ballast, and

timber $= 0{\cdot}3 \times 12 = 3{\cdot}6$; weight of longitudinals $= {\cdot}45$; hence $W = 32 + 3{\cdot}6 + {\cdot}45 = 36{\cdot}05$. If for cross beams, $r = 10$, and $l = 14$, the weight of each will be—

$$W' = \frac{36{\cdot}05 \times 14 \times 10}{1500 \times 4 - 14 \times 10} = {\cdot}86 \text{ tons};$$

or, 0·072 tons per foot run of bridge. Total weight of platform girders $= {\cdot}037 + {\cdot}072 = {\cdot}109$ tons per foot run.

40. *Estimate of weight of main girders.* In dealing with this part of the load, three courses may be pursued:—1st. The weight of the main girders may be assumed from comparison with the weight of similar girders previously constructed, or from tables like those which Mr. Baker has calculated for this purpose. 2nd. The dimensions may be first calculated, neglecting the weight of the main girders, and the weight of the main girders calculated from the dimensions so found. Then the dimensions are to be corrected, so as to include the weight of the main girders, by being increased in the ratio that the strength of the girders as first calculated bears to the excess of strength over weight. This method is proposed by Professor Rankine, and is so fully explained by him that it is not necessary to allude to it further here. 3rd. We may use the approximate formula already given (§35) with constants derived from bridges of similar construction to that the weight of which is required. For this purpose, it will be convenient to substitute the loads per foot run for the total loads.

Let $w_1$ = total live load per foot run of girder in tons.
$w_2$ = weight of platform „ „
$w_3$ = weight of main girder „ „

The rest of the notation remaining as before—

$$w_3 = \frac{(w_1 + w_2)\, l^2}{Cds - l^2} = \frac{(w_1 + w_2)\, lr}{Cs - lr}$$

The following values of $C$ have been deduced from a few well-known bridges:—

| | $C$ |
|---|---|
| Conway, Tubular ... ... ... ... ... ... | 1,700 |
| Britannia ,, ... ... ... ... ... ... | 1,461 |
| Torksey, Tubular Girder ... ... ... | 1,197 |
| Cannon Street, Box Girder ... ... ... | 1,540 |
| ,, Plate Girder ... ... ... | 1,598 |
| Small Plate Girders, 30 to 60 ft. ... ... | 1,280 |
| Connecticut, N Truss ... ... ... ... | 1,548 |
| Charing Cross, Lattice ... ... ... ... | 1,880 |
| Oykell ,, ... ... ... ... | 1,590 |
| Crumlin, Warren ... ... ... ... ... | 1,820 |
| Lough Ken, Bowstring ... ... ... ... | 1,490 |

In Plate II. curves have been drawn, giving the weights of girders for a rolling load of one ton per foot run, and a platform weight of 0·5 tons per foot run, for spans up to 300 feet, and for values of $C = 1{,}200$, $r = 12$; $C = 1{,}500$, $r = 12$, 10 and 8; and $C = 1{,}800$, $r = 10$. The three curves for the same value of $C$ show the influence of the depth of the girder on the weight.

41. *Limiting span of girder bridges.* The formula just given for the weight of bridges shows that the weight of the main girders becomes infinite for a span—

$$l = \frac{Cs}{r}$$

and as the rolling load vanishes in comparison with the dead load, the value to be taken for $s$ is that for the dead load only, when different values are assumed for the dead and live loads. For example, the limiting span will be as follows:—

Case 1. Ordinary tubular bridge; stress on gross section for dead and live load, 4 tons; $r = 12$; $C = 1{,}700$; limiting span 566 feet.

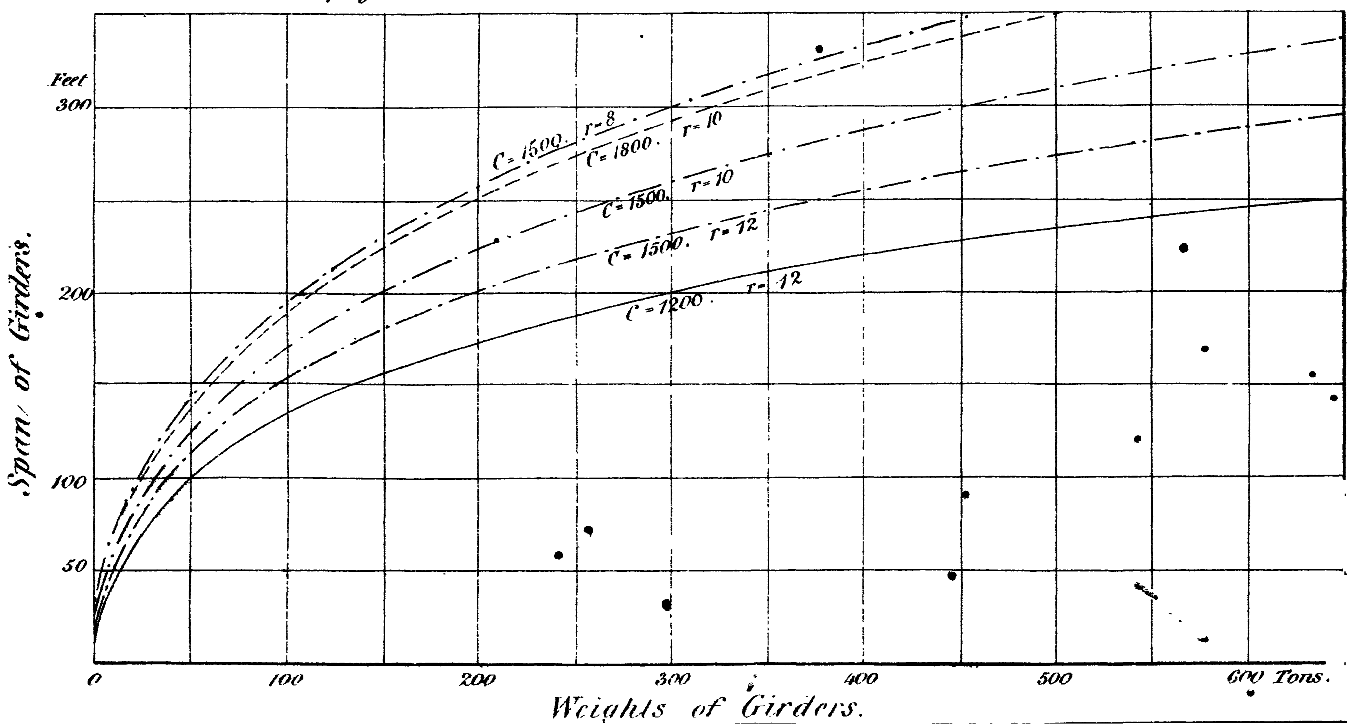
Case 2. Lattice bridge; $s = 4$; $r = 10$; $C = 1{,}900$; limiting span, 760 feet.

Case 3. Tubular bridge as before, but $s$ for dead load $= 7$; limiting span, 992 feet.

Plate II.

# Weight of Main Girders.

Live Load 1 Ton per ft. run. Platform 0·5 Ton. Stress on Gross Section of Booms 4 Tons per square In:



Case 4. Lattice bridge as before, but $s$ for dead load $= 7$; limiting span, 1,330 feet.

The span at which a wrought-iron bridge would break down under its own weight may be obtained by putting $s = 17$.

The practical limit to the span of girder bridges must of course be less than the maximum theoretically possible. But the latter is useful as indicating that the possible span is greater in proportion as the ratio of depth to span increases, and in proportion as a higher limit of stress is permitted, for the stresses due to the dead load. If we suppose the bridge built of steel, $s$ might be taken at 16 tons; and if at the same time the depth of the girder were increased to 1-8th of the span, the theoretical limit to the span would be increased to about 4,000 feet.

*Note on the Resistance of Struts.*

Let $t$ be the average thrust in tons per square inch on the section of a strut; $l$ its length, $h$ its diameter; $f$ the greatest intensity of stress due to thrust and flexure; then Professor Gordon's formula gives

$$t = f \div \left(1 + \frac{l^2}{c\,h^2}\right)$$

for a strut fixed at the ends, and

$$t = f \div \left(1 + \frac{4\,l^2}{c\,h^2}\right)$$

for a strut unfixed at the ends. He gives for the value of the constant $c = 3000$, and for the value of $f$, when the strut is on the point of buckling, 16 tons. Professor Rankine proposes to substitute for $h$ a function of the moment of inertia of the section, but the formula so modified does not agree very well with the few experiments on different forms of section which we possess. Since the value of the constant $c$ must in any case be derived from experiment, for one form of section, the following values so obtained for different forms may be useful. The values of $f$ are for the ultimate

resistance in tons; the working stress should not exceed 1-4th to 1-5th of the values of $f$ here given.

| | $c =$ | $f =$ |
|---|---|---|
| Rectangular Bars ... ... ... | 2500 | 17 |
| Cylindrical Tubes ... ... ... | 3500 | 17·5 |
| Angle, T, Cross, and Channel Iron ... ... ... ... ... | 900 | 19 |

Mr. Davies, of the Crumlin Works, from whose experiments the values for angle iron have been deduced, finds that when the strut is reduced to a flat plate at the ends, for the purpose of rivetting, the strength is reduced by from 1-10th to 1-3rd.

# LECTURE III.

## *PLATE WEB BRIDGES.*

42. Plate or continuous web bridges are those in which the vertical member, the function of which is to resist the shearing force, consists of a continuous rivetted plate, stiffened at intervals by projections of T or L iron. They may be divided into tubular, tubular girder, and plate girder bridges.

The earliest wrought-iron girder bridges, the magnificent structures which cross the Conway river and Menai Straits, are of the tubular type; as also is the great Victoria Bridge in Canada. These bridges, almost the only examples of their class, consist of hollow rectangular girders, through the interior of which the railway passes. The booms, which resist the horizontal tension and compression, form the tops and bottoms of the tubes, and they are thus distinguished from those bridges to which the term tubular is sometimes improperly applied, which consist of pairs of girders united below by a platform and above by overhead bracing. With this restriction it must be said that tubular bridges have been costly, compared with other wrought-iron bridges; and it is probable that even for great spans, to which alone it is applicable, the tubular principle will not again be employed. Having arisen out of peculiar exigencies, tubular bridges may now be considered to have been superseded by more economical structures, which nevertheless owe their creation to these first magnificent exemplifications of a new principle carried out in a new material.

The Britannia Bridge consists of two tubes or tunnels, each of a total length of 1,511 feet, and each carrying one line of way in its interior; it has two principal clear spans of 460 feet, and two of 230 feet. The total depth of the girders at the centre is 30 feet, or 1-15th of the principal span. The total sectional area of metal at the centre of the large spans is, without deducting rivet holes, 648 square inches in the top, 300 inches in the sides, and 585 inches in the bottom. The top and bottom booms present the peculiarity that they are constructed in the form of a layer of rectangular cells. The sides are continuous webs of plate iron, stiffened by vertical T irons at the joints. The tubes were built on shore, floated on pontoons to the foot of the piers, and raised by hydraulic presses to a clear height of 100 feet above the water; a method since adopted by Brunel for the great bridge at Saltash. The tubes are fixed on the centre pier, and on the side piers and abutments rest on a series of rollers, which allow of the free expansion and contraction of the tubes under changes of temperature.

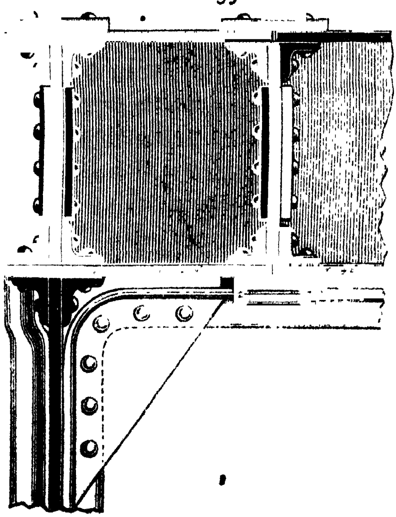
Soon after the erection of the Britannia and Conway Bridges, Mr. Stephenson constructed a tubular bridge of 225 feet span, over the Aire, and boldly abandoned the cellular construction of the booms, which had been considered so essential a part of the tubular system, but which presents grave constructive difficulties in bridges of small span. The same course was followed in the design of the great tubular bridge of 24 spans of 242 feet, and one of 330 feet, erected by Mr. Stephenson over the St. Lawrence, in Canada. None of these bridges, however, are to be commended on the score of economy; they are heavier in proportion to the span than other wrought-iron bridges, and in the case of the Britannia Bridge, had not a high limit of stress and a low estimate of rolling load been assumed, the bridge would probably never have been constructed. In some tubular bridges sent by Mr. Stephenson to Egypt, the railway is carried on the top, instead of in the interior of the tubes, an arrangement which forms a transition from the original tubular system to the admirable tubular girder system introduced by Dr. Fairbairn.

In this latter system, a platform of short transverse girders, carrying the roadway, is laid on the top of or between two or more tubular main girders, whose construction is essentially the same as that of the Britannia Bridge.

43. The modifications derived from the tubular girder with double web and cellular top are—the box beam in which the cells in the top boom are abandoned and the boom made solid, and the plate beam in which the two webs are united into a single plate. Except for large spans, above 150 feet for instance, these forms are undoubtedly better than the more complicated tubular girder; and there is no reason to suppose that (when the tendency to flexure is properly guarded against) they are weaker than the parent form. Some Engineers have expended much ingenuity in devising types in which the arrangement of metal in the top boom is analogous to that in the tops of the first tubular bridges; but it does not appear that such arrangements, confessedly more difficult to manufacture, present any advantage over the simple solid plate boom, at least in those cases in which the dimensions permit us to give to that form suitable proportions.

44. *The cellular system.* The cellular construction of the booms of the Britannia Bridge, shown in Fig. 35, has led to

FIG. 35.



much popular misunderstanding. Probably the *theoretical* importance of this feature was overrated in the earlier history of wrought-iron bridges. The Engineers of the first tubular bridges appear to have been almost startled by the readiness with which the top of the experimental tubes yielded by the vertical bending or buckling of the plates. Long before the iron of these tubes reached its ultimate power of resistance to crushing, the plates bent or crumpled up. The reason of this is not far to seek. Relatively to their other dimensions, these tubes were of extreme thinness; and, in addition to this, the attachment of the booms to the webs by the rivets was, in some of them, very imperfect. In several of these tubes the ratio of length to thickness of top boom was 2,000 to 1 and 3,000 to 1. And although the formula for struts is not directly applicable to the vertical stiffness of booms, because the attachment to the web adds greatly to the vertical resistance to bending, so high a ratio might be expected to reduce materially the ultimate direct thrust, which the boom would carry. When the ratio of length of booms to thickness was reduced to 720 to 1, the tops of the tubes crushed fairly without buckling.

To secure economy in the disposition of the material in the top of a girder, it must be so arranged that its ultimate resistance to crushing is fully called out, and the tendency to flexure eliminated. This may be secured in two ways: either by reducing the width and increasing the thickness of the top boom, or by arranging the plates so that part of them have their width in the direction in which the others tend to buckle. The latter course was the one adopted in the Britannia and Conway Bridges; and hence arose the system of cellular booms, in which the vertical plates of the cells resist the tendency of the horizontal plates to buckle, and *vice versâ.* In a cellular boom the thickness of the boom, as regards resistance to flexure vertically, is practically equal to the depth of the cells.

There is no doubt that this is an admirable method of meeting the difficulty; and in addition to this it had the practical advantage that, until the introduction of drilling,

instead of punching, for forming the rivet holes, it would have been quite impossible to unite the mass of plates required in the booms of large bridges in any other way. Hence, in several bridges, the bottom boom, where no advantage can possibly be derived from the cellular system on the score of strength, has nevertheless been built in cells to facilitate the construction. In small bridges, up to perhaps 150 feet of span, the cells are expensive to construct, and are so small as to be inaccessible for examination and painting. In that case it is much better to abandon the cellular system, and to make the vertical thickness of the solid booms sufficient to resist flexure. Mr. Stephenson abandoned the cells in tubular bridges of 225 and 250 feet span, and Mr. Fairbairn does not use them in girders of less than 100 feet span.

If cells are used, the thickness of the plates composing them must bear a definite relation to their size. The following table of the direct compressive strength of the square cells will show how rapidly the resistance diminishes when the thickness of the plate is less than about 1-30th of the side of the square:—

| Size of cells, in ins. | Thickness of plates, in ins. | Ratio of size to thickness. | Ultimate strength in tons, per sq. in. |
|---|---|---|---|
| 4 | ·03 | 133 | 5 |
| 4 | ·06 | 66 | 8·6 |
| 4 | ·083 | 48 | 11·0 |
| 4 | ·134 | 30 | 10·0 |
| 8 | ·06 | 133 | 5·9 |
| 8 | ·139 | 57 | 9·0 |
| 8 | ·219 | 36 | 11·5 |
| 8 | ·24 | 33 | 12·0* |
| 18 | ·50 | 36 | 13·6 |

That when the boom is of sufficient thickness vertically the cells are unnecessary, may be seen from an experiment of Professor Hodgkinson's, in which a tube or box girder, 45 feet long, 3 feet deep, and 2 feet wide, the top being composed of a plate ¾-inch thick, sustained 17 tons per square

* Not crushed.

inch on the section of the top before it gave way by crushing;* thus sustaining a greater stress than any of the cells quoted above. Looking at the result of this experiment, it would appear that the tendency to vertical buckling is eliminated when the thickness of the solid plate boom is not less than 1-700th of the span. If it be pointed out that in that case the top was firmly connected to very rigid sides, on the other hand, actual girders of the ordinary construction have the advantage of the stiffness due to the angle irons by which they are connected to the web. These angle irons may, with advantage, for booms not cellular, be made as heavy as can be conveniently obtained.

45. *Ratio of width of boom to span.* The width of the boom must be sufficient to resist the tendency to flexure in a horizontal plane, because in that direction it is free to bend. Small I-shaped beams frequently give way, not by the vertical buckling, but by the lateral sagging of the top boom, from deficiency of stiffness in a horizontal plane. So great is this tendency in some cases, that the area of the top boom requires to be augmented to twice the area of the bottom boom, to render its ultimate resistance equal to that of the bottom boom. From some experiments of this kind, Professor Rankine has deduced the following formula for the ultimate strength of the top booms of small girders, as regards resistance to lateral flexure:—†

$$t = \frac{f}{1 + a\,r^2}$$

Where $t$ is the ultimate average thrust per square inch on the section of the boom; $r$ = the ratio of length of boom to width; $f$ = 16 tons; and $a$ is a constant which takes the values

| | |
|---|---|
| For a flat boom ... ... ... ... ... | 1-5,000th. |
| For a square cell ... ... ... ... ... | 1-10,000th. |
| For a cylindrical cell ... ... ... ... | 1-7,500th. |

* Britannia and Conway Bridges.—Edwin Clark.—Vol. I., p. 408.

† Rankine, "Civil Engineering," p. 528.

*In applying this to the booms of the main girders of bridges,* we may substitute $f = 5$ tons, when $t$ should be the maximum safe thrust on the boom. This would give for a safe working stress of 4 tons per square inch a ratio of width of boom to length of 1-35th for plate booms; 1-50th for booms constructed with one square cell. These results agree fairly with actual practice.

In the Britannia Bridge the width of top boom is 1-31st of the span; in tubular girder bridges with cellular tops it varies from 1-50th to 1-80th; in box beams and plate girders with flat top booms, it varies from 1-30th to 1-40th. If in any circumstances these proportions cannot be secured, a lower value must be taken for the limiting stress, or what is the same thing, more metal must be put into the top boom to resist the tendency to flexure. When the top booms of two girders can be braced together, so great a proportion of width is, of course, unnecessary.

46. *Ratio of depth of main girders to span.* In bridges with continuous webs, the ratio of the effective depth to the clear span varies from 1-12th to 1-15th in most cases; indeed, few engineers have ventured to depart much from the proportions adopted for the first tubular bridges.

47. *Method of designing the booms.* In most cases the width of the boom is uniform, and the longitudinal section is thickest at the centre, and tapers towards the ends. Supposing the stress on the boom calculated for distances of about 12 feet along the boom, the maximum cross-section is first designed in accordance with the calculated stress. Next the minimum section is sketched out, and is generally in excess of the requirements of strength, being determined by practical considerations as to the extent to which the thickness of the plates can be reduced, without modification of the general form of section adopted at the centre. The maximum section is then tapered down to the minimum section, in accordance with the calculated stresses.

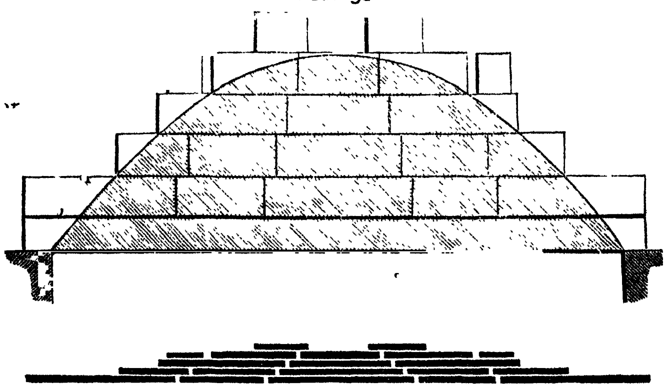
A very convenient plan of proceeding to obtain the longitudinal section of the boom, after the centre cross-section has been designed, is as follows:—Draw the curve of bending

moments (§ 18) and calculate separately the moment of resistance to flexure of each of the plates and bars which make up the centre cross-section. Thus, if $a$ is the sectional area of such a plate; $h$ the effective depth of the girder, or depth from centre to centre of booms; $s$ the limiting stress per unit of section; then the moment of resistance to flexure is:—

$$M = h \ s \ a.$$

At the point corresponding to the centre of the span on the diagram of moments, Fig. 36, and on the same scale, set off vertically upwards the values so found, in order. Through the points thus marked off, draw lines parallel to that representing the span. The intersection of these lines with the curve of moments will give the points at which each layer of plates may terminate. The layers of plates may then be cut

FIG. 36.



up into convenient lengths (say 12 to 15 feet for the bottom boom and 8 to 12 feet for the top boom), and wherever a joint occurs within the curve of moments, a cover plate must be added outside it, equal in thickness to the plate in which the joint occurs. The length of all the plates and the position of the cover plates being thus decided, the longitudinal section of the boom is easily drawn by substituting the actual thicknesses of the plates for their moments of resistance.

Fig. 36 shows a simple plate boom drawn out in this way. The lowest space marked off on the diagram of moments corresponds to the angle irons which connect the boom to

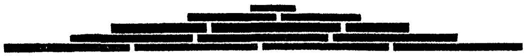
the web. The other spaces are the moments of the successive plates which make up the thickness of the boom. The lower diagram is the actual section of the boom.

In some cases, particularly when the girder is of varying depth, the curve of total stress (§ 25) will be more convenient than the curve of moments for drawing the boom. Then, instead of marking off on the diagram distances equal to the moments of resistance of the plates, the distances must be equal (on the scale of tons to which the curve is drawn) to the products of the sectional areas of the plates and the limiting stress adopted.

48. The following principles are to be attended to in designing the boom. (1) The size of the plates is sometimes limited by the consideration that manufacturers charge a higher rate for plates exceeding a certain size or weight. (The precise rule varies, but about 24 square feet area, or 4 cwt. in weight, is the limit for plates; and 4 cwt. in weight, or 8 inches in the sum of the sides for bars.) (2) The bottom boom is reduced in strength at the joints in proportion to the area cut through or punched out, in any single line across the boom. The joints should therefore be as few as possible, and the plates should break joint so that no two joints occur in the same cross-section. The minimum distance between two joints should not be less than twice the pitch of the rivetting at the joints. The top boom is not weakened to the same extent by the joints, but they are in practice broken in the same manner. (3) The ordinary length of plates is about 12 to 15 feet, according to the width, but they can be obtained, if necessary, up to 30 or 35 feet. Whether the augmentation of cost will be repaid in economy of labour, must be left to the judgment of the engineer in each case. (4) The cross-section must be so designed that the plates can be put together and rivetted up *seriatim*, the rivet heads being accessible on both sides for rivetting. (5) The plates must be as nearly uniform in dimensions as possible, and the rivetting so arranged that the pattern for each plate is as nearly as possible the same. Any departure from this rule involves the risk of mistakes by the workmen.

A form of boom has sometimes been adopted, in which separate cover plates at the joints are dispensed with. Each additional thickness of plates, from the ends to the centre,

FIG. 37.



overlaps a joint in one of the other plates. (Fig. 37.) A somewhat analogous plan is adopted at the Charing Cross Bridge, where, in the horizontal tables of the booms, an additional layer of plates is introduced, running the whole length of the girder, and serving to cover all the joints in the other plates. The area of this plate is, of course, omitted in calculating the strength of the bridge. As the booms in the Charing Cross Bridge are of enormous strength, and the plates only $\frac{5}{8}$-inch thick, the joints are so numerous that no great loss of material is involved in the substitution of a continuous for a discontinuous cover plate.

45. *Method of designing the webs.* The sides or webs of continuous web girders consist of vertical plates, placed side by side, and connected by a covering plate on one side and a T iron on the other, or by a T iron on both sides. The object of the T irons is to stiffen the web to resist the diagonal compressive forces (§ 27) to which the shearing force gives rise. It has already been shown that the shearing force may be considered to be resisted by the web alone, and that it produces tangential forces of equal intensity on vertical and horizontal sections, which are equivalent to forces of tension and compression acting at right angles to each other, along lines inclined at 45° to the axis of the beam.

In proportioning the web, the first point to be attended to is that the area of metal, in sections through the vertical and horizontal joints, should be sufficient to resist the shearing force. Let $F$ be the shearing force on the web, if single, or on each web, if double, at any vertical section of a beam; $h$ the effective depth of the girder in feet; and $s$ the limiting safe shearing stress per unit of net area, which is usually taken at 4 tons per square inch. Then the shear *per foot run* of a

*vertical joint at the given point of the beam, or of a horizontal joint in the neighbourhood of that point, will be $F \div h$.* The section of metal required to resist the shear will be, per foot run of joint,

$$a = \frac{F}{h\ s}$$

This will be the required section: (1) through the line of fracture of the plates; (2) through the line of fracture of the cover plates; and (3), through the rivets. Suppose there are $n$ rivets, $d$ inches diameter, per foot run, and that the web plates are $t$ inches thick. The area punched out of the web plate is $n\ d\ t$, and the section left is $(12 - n\ d)\ t$, which must be not less than $a$. The shearing section through the rivets will be $\cdot 785\ n\ d^2$ if the joint is covered on one side only, and $1 \cdot 57\ n\ d^2$ if the joint is covered on both sides, which also must be not less than $a$. The joint is supposed to be rightly proportioned in other respects, as will be described presently.

To support the diagonal compressive force acting in a direction downwards towards the nearest abutment (in uniformly loaded bridges), and inclined at an angle of 45° with the axis of the beam, the web must be strong enough, as a pillar, to resist a thrust of $F \div h$ tons per foot run of width, measured at right angles to that direction. Suppose, first, there are no stiffening pillars, or that the stiffening pillars are at a greater distance apart than the depth of the girder, then the length of the pillar resisting the thrust is the distance between the booms, measured along a line inclined at 45° with the axis, or a little less than $h \sqrt{2}$ feet. The ratio of the length of the pillar to its least diameter is

$$\frac{12\ h \sqrt{2}}{t}$$

and its sectional area in square inches for one foot width is $12\ t$. Substituting these values in the formula for the resistance of columns, and taking the limiting stress at 4 tons, we get

$$T = \frac{48\ t}{1 + \frac{h^2}{9\ t^2}}$$

for the safe thrust per foot width. Then $t$ must be so taken that $T$ is greater than $F \div h$.

If there are stiffening pillars to the web, the length of column resisting the thrust is reduced to that of a line measured at 45° with the axis between two stiffening pillars. So that if $d$ is the distance in feet between two such pillars, $d$ must be substituted for $h$ in the above equation. The value of $T$ may be increased, if necessary, either by reducing $d$ or increasing $t$.

Since the shearing force is greatest at the abutment, the web is first designed at that point; next the centre section is designed, for which the plates are generally not less than $\frac{5}{16}$-inch in moderate-sized bridges, and $\frac{3}{8}$-inch in large bridges. It will then be easy to graduate the section of the web, examining its strength at those points at which the thickness changes.

50. *Stiffening pillars.* The T iron stiffening pillars have a double duty to perform. By their transverse resistance to flexure they oppose the tendency of the web to buckling; and they receive usually the weight brought on to the main girder by the cross beams, and distribute it uniformly over a vertical section of the web, which in turn transmits it to the booms. There must therefore be a stiffening pillar at the junction of each cross girder with the main girder, and there are generally intermediate pillars of somewhat less section to stiffen the web.

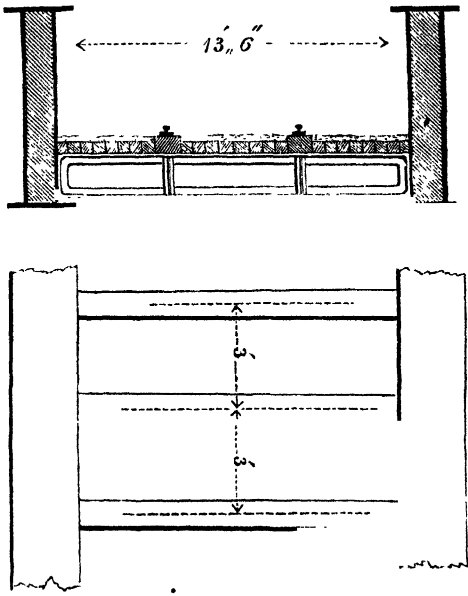
51. *End pillars.* The web at the ends of the girder where it rests on the piers or abutments, is to be considered as a simple pillar, which receives and distributes to the web the entire vertical reaction at that point. It will therefore generally be found that over the piers the web requires an addition to its thickness and an increase in the number of T irons rivetted to it.

## CONSTRUCTION OF PLATFORM.

52. In the earlier wrought-iron girder bridges cross-beams were employed to carry the roadway, placed at sleeper distance or about 3 feet apart. (Fig. 38.) Over these were

placed the longitudinal timbers supporting the rails. It was assumed that these timbers were sufficiently strong to distribute the load brought on them, and the cross-beams were calculated to sustain the same average load as the main girders.

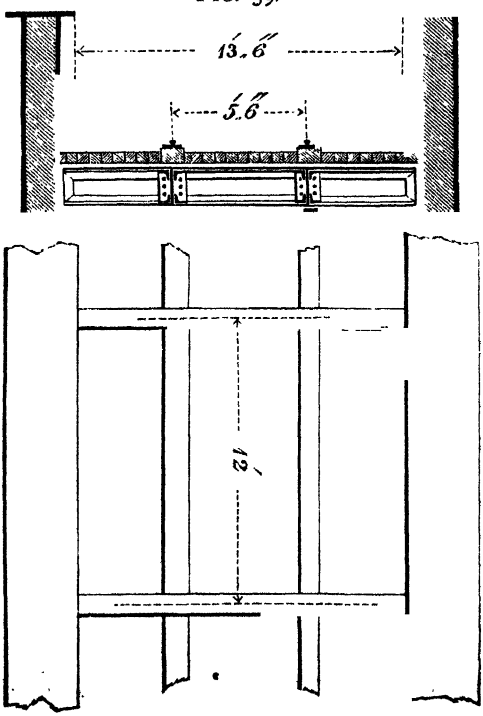
FIG. 38.




A fatal accident, however, to some cast-iron girders, at last aroused the attention of Engineers to the fact that the cross-beams were liable to be strained much more severely than had been assumed. The weight of a locomotive is not carried equally by all its wheels, and the greatest weight on any pair of wheels is brought in turn over each cross-girder. Even if the longitudinal timbers acted as beams, and distributed the load, they could only do so in proportion to the deflection of the girder immediately under the load, which would even in that case have to carry more than the average rolling load of the bridge. But the longitudinal timbers decay; they are not continuous, but have joints, and joints in the timber and rail may occur over a cross-girder. In any of these cases the

cross-beam has to carry the whole load brought on it by the engine wheels, which will be greatly in excess of the average load for which the main girders are calculated. Hence inspectors of railways are now accustomed to examine very narrowly the strength of cross-beams, and they have laid down the following principles to be observed in their construction: (1) The rails, timbers, and sheeting cannot be considered to form part of the structure of the bridge. They are perishable, and are replaced from time to time by men incapable of judging of their functions as part of the bridge, and, consequently, if relied on for strength their repair may involve the safety of the bridge. (2) If the cross-beams are more than 3 feet, or 3 feet 6 inches apart, iron longitudinal girders must be placed under the rails. (3) The rails and sleepers cannot be considered to distribute the load. Every cross-beam must therefore be strong enough to carry the

FIG. 39.



maximum load which will in any circumstances be concentrated over a space equal to the distance between two cross-beams.

Under these regulations, and with the peculiar conditions of loading already discussed (§38), it soon became apparent that the platform would be lighter if the distance between the cross-beams were increased, and longitudinal girders placed under each rail, as shown in Fig. 39, than if the old plan of cross-beams at sleeper-distance apart were retained. I believe that Mr. W. Anderson first pointed out the great economy in weight of the new construction,* as shown in the following table prepared by him. The maximum weight of engine is assumed at 34 tons; maximum load on driving wheels, 16 tons; wheel base, 12 feet. Depth of beams, 1-12th of the span.

| | Span. Feet. | Total Load. Tons. | Net area of bottom flange. ins. | Weight of beams. lbs. | Weight per ft. run of Bridge. |
|---|---|---|---|---|---|
| SINGLE LINE. | | | | | |
| Cross-beams, 3 feet apart. | 14 | 17.26 | 5.3 | 1206 | 402 |
| Cross-beams, 12 feet „ | 14 | 29.35 | 10.9 | 1700 | 268 |
| Longitudinals ............... | 12 | 19.54 | 10.8 | 1518 | |
| DOUBLE LINE. | | | | | |
| Cross-beams, 3 feet apart. | $25\frac{1}{2}$ | 35.00 | 11.4 | 3555 | 1218 |
| Cross-beams, 12 feet „ | $25\frac{1}{2}$ | 58.64 | 19.2 | 4704 | 645 |
| Longitudinals ............... | 12 | 38.64 | 21.6 | 3026 | |

53. *Position of platform in relation to main girders.* The platform in girder bridges may be on the top, between or underneath the main girders. When there is no restriction as to headway under the bridge, it is best placed over the main girders, both because the load is then transmitted fairly to the centre of the main girders, and because the span of the cross-beams may be reduced and the weight correspondingly lightened. If there are two main girders to each line of way, the best position for them is immediately under the rails, when the cross-beams will only have to support the roadway,

* Proc. of Inst. of Civil Engineers of Ireland, 1866.

and may even in some cases be replaced by wood beams. When there are three main girders to two lines of way, they may be placed so that they are equally loaded. Thus if the gauge of the rails is 5 feet 6 inches, and the space between the two ways 6 feet 3 inches, the main girders will be equally loaded if spaced at 8 feet 9¾ inches, and the roadway placed symmetrically over them. If, on the contrary, the main girders are spaced at 11 feet 9 inches apart, centre to centre, the centre girder will support one-half and each outside girder one-fourth of the external load.

When the headway under the bridge is limited, the cross-beams are placed at the bottom of the main girders, and most commonly rest on the bottom boom, and are rivetted to the stiffening pillars of the web. In single web bridges it is advisable to add to the stiffening pillar a plate gusset, at the junction of the cross-beams, which may sometimes extend the whole depth of the web. In double web bridges, in order to transmit the load of the cross-girder to both webs, plates must be introduced between the webs rivetted to the opposite T irons. Sometimes the cross-beams are placed underneath the main girders, and attached to the bottom boom by bolts, near its centre, so as to throw the load equally on both webs. The only objection to this plan is that the projecting ends of the cross-beams break the fair bottom line of the bridge. The ends of the cross-beams are sometimes covered with ornamental cast-iron plates, and these might be made continuous if necessary. If one or two lines of way are carried between two main girders, each takes half the load. If two ways are carried between three main girders, the outside girders take each one-fourth and the centre girder one-half the load.

In all cases the relative proportion of the load borne by the main girders may be ascertained by considering that the loads on the main girders are equal to the supporting forces of the cross-beams.

54. *Construction of cross-beams.* The cross-beams and longitudinals are almost always simple I-shaped beams. Fig. 40 shows a section of a cross-beam with its longitudinal attached,

FIG. 40.

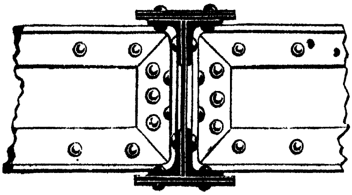
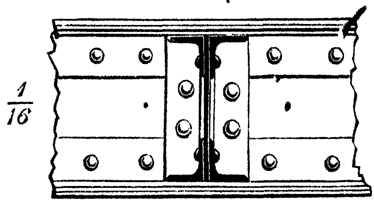


FIG. 41.




suitable for the platform of a bridge with one line of way between the main girders. Fig. 42 shows a section of a similar cross-beam with longitudinal attached for carrying two lines of way. Sometimes it becomes of great importance to restrict the thickness of the platform. In that case the cross-beams may be, as shown in Fig. 43, reduced to a

FIG. 42.

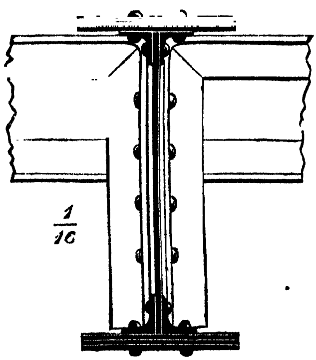
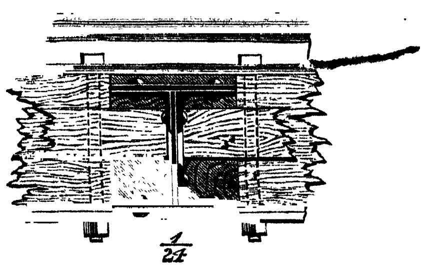
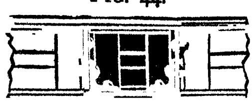



FIG. 43.




minimum depth of 9 or 10 inches, for 14 feet span, spaced 3 feet apart, and carry the longitudinal sleepers in short lengths between them. In this arrangement the longitudinal girders are dispensed with or are placed on top of the cross-girders, and spaced 7 feet apart, so that they practically reduce the span of the cross-girders to that distance. A better arrangement is to form the longitudinals of trough section, as shown in Fig. 44, and to place the longitudinal sleepers which carry the rails within them. The cross-beams and longitudinals have usually a depth of 1-12th of the span, but that proportion

FIG. 44.



may be reduced to 1-16th to save headway, or increased to 1-10th to save weight, according to the judgment of the Engineer.

55. *Advantages of plate web girders.* Leaving for the present the question of the relative economy of plate web as compared with other girders, the following points of difference may be noted. (1) A superiority has been claimed for the continuous web on the score of rigidity. It has, however, been shown by Mr. Stoney,* who has drawn the deflection curve of braced girders to a highly-exaggerated scale, that the construction of the web has scarcely any influence on the deflection, which is due almost exclusively to the lengthening and shortening of the two booms, and which must therefore be the same whatever the construction of the girder, so long as the proportions are the same. Hence, even for girders of the same depth, the deflection of braced girders should be nearly the same as that of continuous web girders. But on the other hand, braced girders are usually deeper than plate girders, and as the deflection is inversely as the depth, we might expect to find some braced girders more rigid than plate girders. For instance, the deflection with proof load of the Victoria Bridge, the latest and strongest of the tubular bridges, was 1-1920th of the span. That of the Boyne lattice bridge was only 1-2110th. That of the Passau lattice bridge was 1-2120th. These are bridges of about the same span, and the braced girders deflect least. (2) The web of continuous girders carries some part of the horizontal stresses, and to that extent aids the booms. The advantage in this respect is equivalent to the addition of 1-6th of the net area of the web to the booms. (3) In the original tubular girders there is enormous horizontal rigidity to resist the action of the wind, but on the other hand that type has the disadvantage that towards the ends there is a greater excess of material in the booms, above that required for strength, than in other forms.

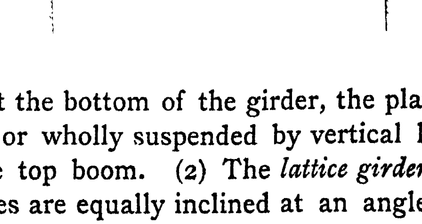
* Transactions of the Royal Irish Acadamy, 1864.

# LECTURE IV.

## *ON BRACED GIRDERS.*

56. Of girders with a discontinuous web, composed of struts and ties arranged according to a system of bracing, some are of uniform depth, and in others one or both booms are curved. In the former class are included: (1) The *Warren girder*, in which the struts and ties are equally inclined at an angle of 60° with the horizontal, and which usually has one system of triangulation, but sometimes two. Fig. 45 shows two arrangements of the Warren girder. When

FIG. 45.

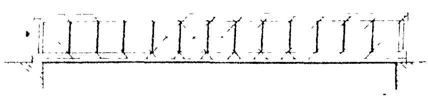


the load is at the bottom of the girder, the platform is sometimes partly or wholly suspended by vertical links from the joints of the top boom. (2) The *lattice girder*, in which the struts and ties are equally inclined at an angle of about 45° with the horizontal (Fig. 46), and which has two or more

FIG. 46.

systems of triangulation. When the bracing bars are very numerous, so as to divide the web into a great number of small reticulations, the girder is called a *trellis* girder. (3) The *Whipple-Murphy truss* or girder, in which the struts are vertical and the ties inclined at from 35° to 45° with the horizontal (Fig. 47). The simplest method of explaining the

FIG. 47.



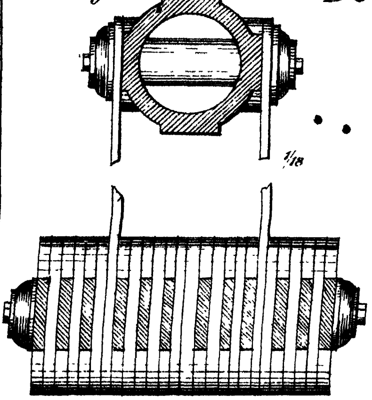
peculiarities of these types of braced girder will be to describe briefly a few well-known bridges.

57. *Warren girder bridges.* The finest examples of the Warren girder in this country are the Newark Dyke bridge on the Great Northern Railway, and the Crumlin Viaduct in South Wales. The Newark Dyke bridge is [illegible]½ feet clear span, and each line of way is carried by a pair of main girders, the roadway being carried on cross-beams resting on the bottom booms of the main girders. The top boom is of cast iron, and the bottom boom consists of a chain of links set vertically, the section of the girder resembling that of a later bridge of similar construction, shown in Plate III., Fig. 1. The struts are of cast iron, of cross-shaped section.

The high compressive resistance of cast iron (from 30 to 45 tons per square inch), and the facility of casting it in the forms most suitable to resist the thrust, mark it out as beyond comparison the best material for the compression members of a girder, but for two fatal defects. The first of these is its extremely small *range* of elasticity, which involves a low power of resistance to sudden or impulsive strains. The other defect is that in its elongation and permanent set under stress, and its expansion from changes of temperature, it follows widely different laws from the wrought iron with which it has to be associated. Under a purely statical load its use is unobjectionable in most cases; but in bridges in

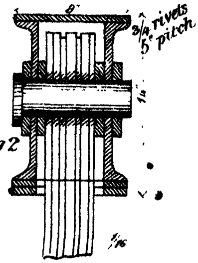
# BOOMS.

Fig. 1




Queensland

Fig 2




Crumlin

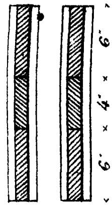
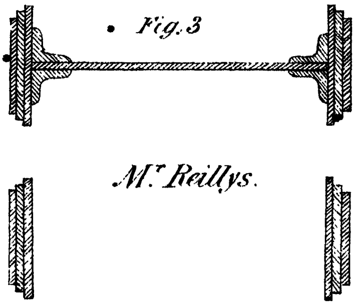
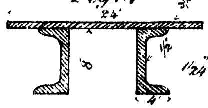



Fig. 3



M^r Reillys.

Fig. 4




Lough Ken.

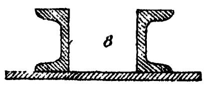
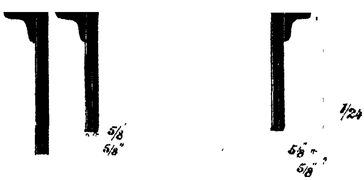



Fig. 5.

6/8" plates: 1" rivets. 4" pitch




Charing Cross.

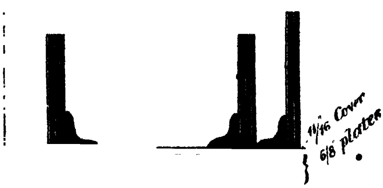
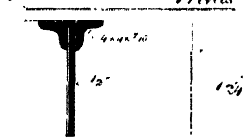
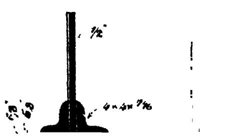



Fig. 6.

1" rivets




Connecticut.



which the load changes, which are subject to concussion, vibration, and other dynamical conditions of strain, and in railway bridges especially, in which an engine off the line from the snapping of a rail is not an unknown contingency, the use of cast iron in combination with wrought iron is seriously to be deprecated. If used at all, a large factor of safety must be adopted. Thus, in the Newark Dyke bridge the cast-iron portions carry a stress of only 1-9th of their ultimate resistance; while the wrought-iron portions are strained to 1-5th. Nevertheless, much as the use of cast iron is discountenanced by English Engineers, it must be admitted that it is largely used in America in braced girders, and apparently with economy and success. In some later bridges on the Newark Dyke pattern, the cast iron has been retained in the top boom, but the struts and all other portions of the bridge have been of wrought iron. A glance at Fig. 1, Plate III., will show that girders of this type have very little lateral stability, and that they require a complete system of bracing between the top booms of a pair of girders to render them secure.

The Crumlin Viaduct, which carries the Taff Vale Railway, at a level of 210 feet above the valley of the Ebbw, is divided into two sections, one of ten, the other of three spans, by a spur from the hills. The extremely light appearance of the Warren girders of this bridge, carrying the railway on their top, supported by lofty piers composed of clustered cast-iron columns, and backed by finely-wooded hills, renders this bridge one of the most beautiful iron constructions in the world. The top booms are rectangular cells (Plate III., Fig. 2), the bottom booms consist of chains of wrought-iron plates set vertically and rivetted together. The ties are flat links, and the struts are cross-shaped (Plate IV., Fig. 3), and composed of wrought-iron plates and angle bars rivetted together. The bracing is connected to the booms by pins 3 to 3¼ inches in diameter. Since the original construction of the bridge, it has been found necessary to supplement the connection of the bracing bars with the bottom boom by rivetted gusset plates. The width of the booms of the

Crumlin girders is only 9 inches, or 1-200th of the span. It might have been expected, therefore, that they would show a want of lateral stability. It has, in fact, been found desirable to connect the four booms of the parallel girders by a complete deck or platform of iron plates, so that they now form parts of a horizontal as well as of a vertical girder. It ought to be said, in part explanation of the fact that this beautiful structure has required extensive strengthening since its erection, that it carries a much heavier traffic than that for which it was designed.

58. *Lattice bridges.* The Boyne bridge, on the Dublin and Belfast Railway, may be instanced as a fine example of a lattice bridge. It consists of three spans, a centre span of 264 feet, and two side spans of 138 feet 8 inches, the girders being continuous over the three spans. The railway is carried on cross-beams resting on the bottom boom. The booms are of the trough-shaped section (Plate III., Figs. 4, 6), which is now the most usual form of boom in braced girders. The web is double, one web being rivetted to each vertical plate of the trough, but the corresponding struts are braced together so as to act, as regards flexure at right angles to the plane of the girder, as a single strut. (Plate IV., Fig. 6.) One of these struts sustained 14 tons per square inch when tested experimentally. The ties are simple plate-iron bars, $\frac{5}{8}$-inch thick and $4\frac{1}{2}$ to $10\frac{1}{2}$ inches wide, rivetted to the outside of the vertical plates of the booms, and to the struts at their intersection.

In the earlier lattice bridges, advantage was supposed to be derived from the use of a great number of bracing bars, cutting up the web into small reticulations, the bars being rivetted together at every intersection, so as to approximate to the condition of a plate web. This arrangement of the lattice-web permits the use of common descriptions of rolled iron, such as channel and T iron, to form the struts in very large bridges. The rivetting of the struts to the ties undoubtedly stiffens them against flexure in the plane of the girder. At the same time, the multiplication of their number and the reduction of their individual dimensions, both weakens them as regards trans-

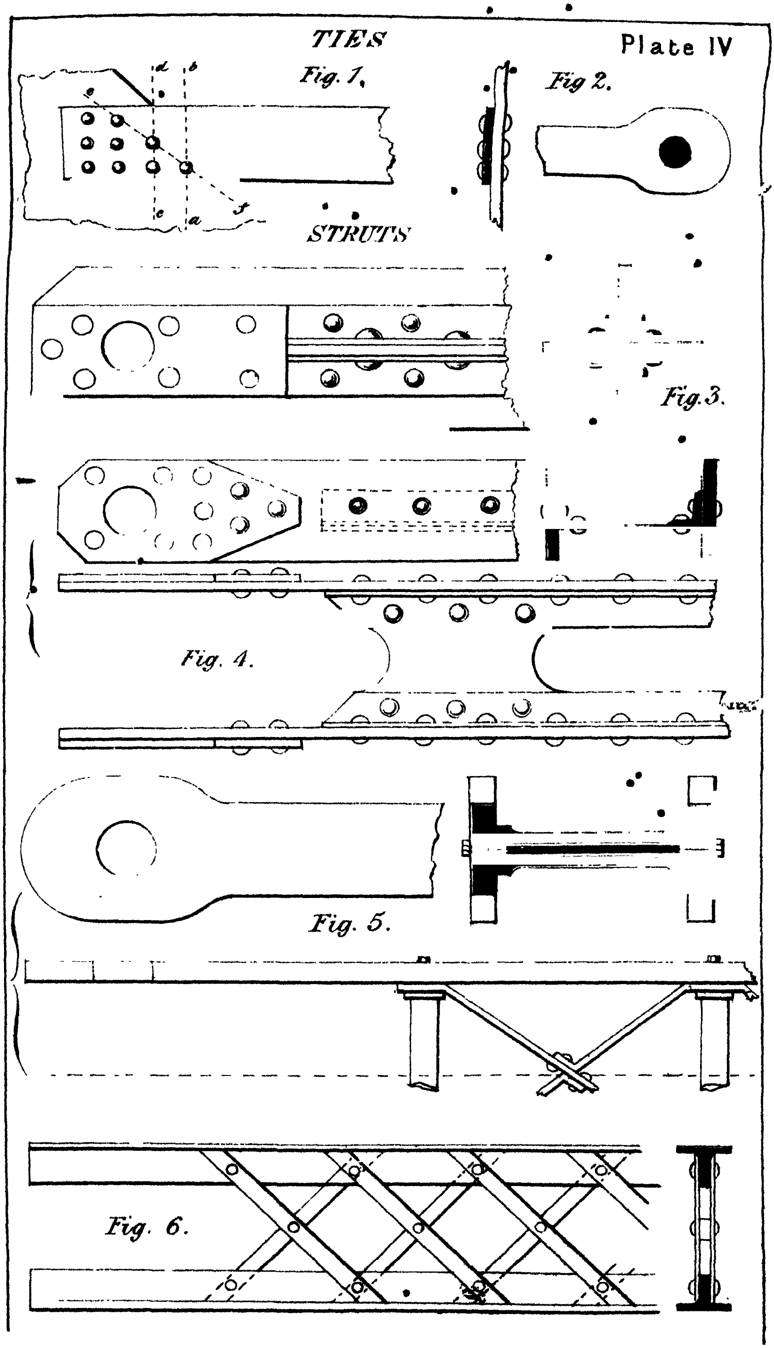


Plate IV
TIES
Fig. 1.
a
b
c
d
e
f
Fig 2.
STRUTS
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.

verse flexure, and increases the difficulty of effecting a sound connection by bracing between a pair of corresponding and parallel struts. In addition, the less the dimensions of the individual struts and ties, the greater will be the proportionate waste of material towards the centre of the girder where the shearing force vanishes; for in general they cannot practically be reduced below certain fixed dimensions.

Hence, in the latest and best practice the number of systems of triangles in the bracing is reduced as much as possible, the limit being always the practical conditions which fix the maximum size of a single strut. The Charing Cross bridge is a noble example of the most perfect type of the lattice girder. The principal spans of this bridge are 154 feet in the clear. There are two main girders over each span, with cross-beams attached beneath them, forming a platform carrying four lines of way. The top and bottom booms (Plate III., Fig. 5) are 4 and 3 feet wide respectively, having each four vertical ribs or feathers. There are five thicknesses of $\frac{5}{8}$-inch plates in the tables of the booms. All the rivet holes are drilled, the rivets being 1 inch in diameter and 4 inches pitch. The diagonal ties are composed of Howard's patent rolled suspension links (Plate IV., Fig. 2). The struts consist of solid forged bars, the corresponding bars being connected together by bolts passing through cast-iron distance pieces (Plate IV., Fig. 5), and by light diagonal bracing. The end pillars over the abutments are in the form of plate-iron boxes, the sides being stiffened with T irons. The connection between the bracing and booms is by puddled steel pins, varying from 5 to 7 inches in diameter.

59. *Whipple-Murphy truss.* In the two preceding forms of braced girders the ties and struts are of equal length. If the same section of metal were required to resist tensile and compressive forces, then it can be shown that the lattice girder with bars inclined at 45° would be the most economical form of braced girder. But this is not exactly the case in practice. In the top boom the tendency to flexure can be so far eliminated that the average stress on the gross section may be nearly or quite as high as in the bottom boom, and in

many bridges the top and bottom booms are precisely similar; but in the struts the transverse dimensions bear a less ratio to the length, and the safe stress is less on the struts than on the ties, or at all events bracing must be supplied to stiffen the strut which does not count in its section. In well-designed bridges the struts are from 1-5th to 1-10th heavier than the ties for equal stresses, the excess of weight, however, increasing in a high ratio with the length of the strut. It appeared, therefore, to an American Engineer that the best position for the strut would be that in which its length was least. Hence arose the Whipple-Murphy or N-truss, in which the struts are vertical and the ties inclined at angles of from 35° to 40° with the horizontal. This form of girder has been largely used in America, and several examples have been erected in this country.

The Connecticut River bridge on this system was constructed in this country, and sent to the States during the American war. It consists of 17 spans, the largest of which are 138 and 177 feet in the clear. The bridge carries a single line of way on the top of the main girders, 16 feet apart from centre to centre. The booms are 25 inches wide (Plate III., Fig. 6), and are composed of plates from $\frac{3}{8}$-inch to $\frac{3}{4}$-inch thick, and generally 17 feet in length. The struts consist of T iron uprights rivetted inside the vertical plates of the booms, the opposite T irons being braced together; the T irons vary from $4\frac{1}{2} \times 3\frac{1}{2}$ inches $\times$ $\frac{3}{8}$-inch to $6 \times 4$ inches $\times$ $\frac{5}{8}$-inch. The diagonal ties which, except at the ends, cross and are rivetted to the struts at their centre, are simple plate-iron bars, varying from 9 inches $\times$ $\frac{5}{8}$-inch to $2\frac{1}{2}$ inches $\times$ $\frac{1}{2}$-inch. The largest of the ties are connected to the booms by eleven 1-inch rivets at each end, giving a large shearing and bearing area compared with the section of the tie. There is a peculiarity in the way the counterbracing is carried out, which merits attention. In place of strengthening the ties near the centre of the bridge, an additional tie is provided, sloping in the opposite direction to that it should have for a uniformly distributed load. Of the two ties in the centre panels (Fig. 47), one or other acts according to the point at

which the shearing force changes sign under the influence of the rolling load. The other, which from its direction would be a strut, being incapable from its form of carrying any considerable compression, bends and allows the vertical struts to resist the thrust in all positions of the rolling load.

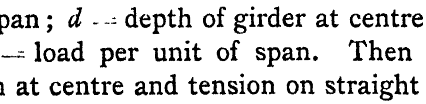
As further examples of this system, some bridges designed by Mr. Calcott Reilly may be alluded to. Mr. Reilly noticed that with the ordinary trough-shaped boom the point of attachment of the bracing was not at the centre of gravity of the section of the boom. Hence, considering a cross-section of the boom between two consecutive joints, and neglecting the deflection of the boom, the stress brought on the boom at the joints would not be uniformly distributed over the section. To take a rough illustration, if a prism of indian-rubber is compressed by forces acting at the centres of two rigid plates, at its ends it will be uniformly compressed. But if the points of application of the pressures do not coincide with the centres of the ends, but are nearer one side than the other, the prism will be more compressed on that side than on the other. This is what Mr. Reilly supposes to happen in a trough-shaped boom, when the centres of the joints are below the centres of the booms.* Fortunately the curvature of the booms by deflection tends to produce an inequality in the distribution of the stress in the opposite direction to that due to the eccentricity of the joints of the bracing bars, and the two inequalities tend to compensate each other. Mr. Reilly, however, was led to adopt some forms for his booms, in which the joints of the bracing are at the centre of gravity of the cross-section, and which in other respects are both new and meritorious. In this form of boom the great mass of the plates is placed vertically, instead of horizontally (Plate III., Fig. 3), in two portions. In the top boom these are connected by a horizontal plate, so that the section is H-shaped, or in the form of an ordinary girder laid on its side. Both the horizontal and vertical axes of the booms are axes of symmetry. The bracing is attached by bolts through the centre of the section, the horizontal

* Proceedings of the Inst. of Civil Engineers, 1864-5, p. 39.

plate of the top boom being stopped off for a short distance at the joints. This form of boom presents the great advantage that it is much stronger to resist lateral flexure than any other form.

60. *Bowstring bridges.* When the top boom of a girder is arched and the bottom boom straight, the girder is called a *bowstring* girder; or if the lower boom is arched and the top boom straight, an *inverted bowstring* girder. If either boom be arched it shares with the web the shearing force, which in a parallel girder falls wholly on the latter. If the curve of the boom is parabolic—and for the proportion of versed sine to span usual in bridges, a circular arc does not sensibly differ from a parabola—then with a load uniformly distributed over the span, the entire shearing force at any section is balanced by the vertical component of the stress on the boom, and the girder does not require any bracing. If a bridge be required to carry a canal aqueduct or other uniform load, then the girder may consist of a parabolic arched boom, a horizontal tie, and vertical rods to transmit the load to the

FIG. 48.



arched rib. In such a bridge the stresses are given by the following formulæ—*

Let $l$ = span; $d$ = depth of girder at centre; $n$ = number of bays; $w$ = load per unit of span. Then the thrust on arched boom at centre and tension on straight boom thoughout is—

$$H_1 = wl^2 \div 8d \quad \ldots \quad \ldots \quad (1)$$

The thrust at any point of the curved boom at a horizontal distance $x$ from the centre, on a section normal to the curve, is—

$$H_2 = \sqrt{(H_1^2 + w^2 x^2)} \ \ldots \ (2)$$

The tension on each of the suspending rods is:—

$$t = wl \div n \quad \ldots \quad \ldots \quad \ldots \ (3)$$

* Rankine, "Civil Engineering," p. 563.

If the girder is an inverted bowstring, the same formulæ apply, writing tension for thrust and thrust for tension.

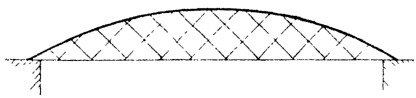
The great practical advantage of the bowstring over other forms of girder results from the fact that the stress on its members is nearly uniform from the centre to the abutment. There is a difficulty in girders of uniform depth in realising the best distribution of the material, owing to the great variation in the magnitude of the stresses. But in the bowstring bridge there is no great variation in the stress, and consequently there need be little variation in the section of the parts. With a uniform load the tension on the horizontal boom is uniform, that on the ties is uniform, and that on the arched boom increases from the centre to the springing by only 8 per cent. if the ratio of span to depth is 10; and by 12 per cent. if the ratio is 8. Hence if the section of all the parts were made uniform throughout, the excess of material above that theoretically necessary need not be more than 2 or 3 per cent. of the weight of the girder. In such a bridge, therefore, the bars and plates might all be of uniform dimensions from end to end of the span. The advantage of this from a practical point of view can scarcely be overrated. Lastly, as this form of bridge easily admits a depth of 1-8th of the span, we shall be prepared to find that it is a very economical construction.

61. *Bowstring bridge with travelling load.* When a bowstring bridge is subject to a rolling load a new difficulty arises—the shearing force is no longer wholly transmitted through the boom. With a partial load covering the longer of two segments, into which a section divides the bridge, the vertical component of the stress on the arched boom is less and the shearing force is greater than with a uniform load. The excess must be resisted either (1) by the stiffness of the arched rib, or (2) by bracing between the arched and straight boom, or (3) by independent girders braced to the arch. The first method is that followed in the construction of cast iron, the second of wrought iron, and the third of timber bridges. For wrought-iron bridges the most common arrangement is that shown in Fig. 49, where the bracing consists of vertical

FIG. 49.

struts and diagonal ties. A better plan, as sacrificing less the uniformity of section in the parts which is the merit of the system, is shown in Fig. 50. A single system of triangu-

FIG. 50.



lation may be adopted, or several if necessary, of precisely the same kind as is usual in girders of uniform depth.

62. As an example of a bowstring bridge, the Lough Ken Viaduct, on the Port Patrick Railway, may be mentioned. This bridge has spans of 130 feet. The booms are trough-shaped (Plate III., Fig. 4), and of uniform section throughout. The ties are flat bars, and the struts of the braced form common in lattice bridges.

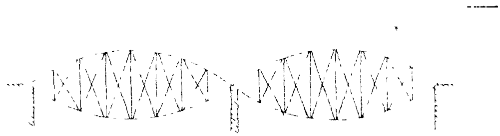
In Sweden the inverted bowstring has been used by Major Adelskold for a steel bridge of 137 feet span. The resistance of soft steel in compression is inferior to its resistance in tension, because with the reduced dimensions required in that material, as compared with iron, flexure is less perfectly prevented. Major Adelskold, therefore, judged that in a structure in which economy of weight was of the highest importance, the shorter or horizontal member should be in compression.

63. Many systems of bridges, differing more or less widely from those already described, have attracted attention. Excluding from consideration those which are not strictly girders, there remain three forms requiring a brief description.

There is, first, the *bowstring suspension* bridge, or bowstring bridge with both booms arched. The great bridge erected by Mr. Brunel at Saltash, with spans of 445 feet, is of this con-

struction, and also the bridge over the Rhine at Maintz, with spans of 326 feet. Fig. 51 will give an idea of the principle

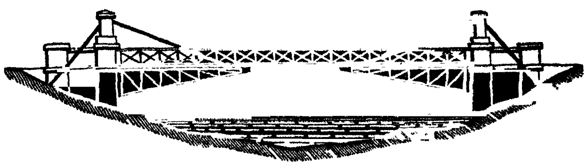
FIG. 51.



of these bridges. By adopting a suitable curvature in bridges of this form, it is possible to secure, that the stress on any normal section of the booms shall be uniform througout the span—a point of importance in large bridges. But the depth of the girder would not in that case diminish to zero at the abutments. The ratio of depth to span in these bridges reaches 1-7th in the Maintz and 1-8th in the Saltash Bridge. As in the bowstring bridge, the bracing is very light, being only required to transmit the load at each point to the booms and to resist the distortion due to partial loading.

64. In Fig. 52 is shown the *cantilever* bridge of Mr. Sedley. Since the weight of the main girders of a bridge per foot run

FIG. 52.



of length increases as the square of the span nearly,* a great saving will be effected in bridging a given distance, by any arrangement which is tantamount, in respect of the sections of metal required in the girders, to a reduction of span. This is the source of the economy arising from continuity of the girders over several spans, because in that case, so far as the stresses on the booms are concerned, the continuous girder over $n$ spans is equivalent to $2n$-1 separate girders, each of

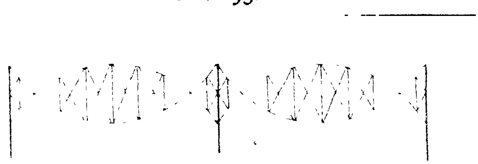
* That is if the depth is constant. If the depth is variable and the ratio of depth to span constant, the weight per foot run is as the span nearly (§ 40).

proportionately less length and less weight. Mr. Sedley accomplishes a reduction of effective span in another way. He supports the bridge at two points by suspension chains, so as to divide the span into three nearly equal portions. The segments next the abutments are cantilevers partly encastré and partly supported at the other end by the chains. A continuous girder is then constructed, attached to the cantilevers and braced to the chains.

65. If a bridge on Mr. Sedley's principle be taken, and the chains dispensed with, in order to do which the cantilevers must be rendered completely encastré with the piers,* we shall in reality get a bridge on the systen of Von Ruppert, which attracted much attention at the Paris Exhibition, and with which he proposes to cross spans hitherto unattempted in girder bridges. In this system the girders are continuous with each other, and the superstructure is rigidly united with the substructure.

The bowstring suspension bridge, in its ordinary form, does not admit of continuity between the girders or between the girders and the piers. If, however, the curved booms (Fig. 51) be caused to approach each other (Fig. 53), we get Von

FIG. 53.



Ruppert's modification, obtaining at once the advantage of the reduction of effective span from the continuity of the girders and their connexion with the piers. An examination of the theory of these bridges will be found in the "Engineer."†

66. *Relative advantages of different forms of girder.* It would appear at first sight extremely simple to determine by exact calculation which form of girder required the least material;

* Mr. Sedley appears to have effected a modification of this kind in later bridges.

† Vol. XXIV., p. 221, *et seq.*

and if girders could be constructed with the sectional area of the parts exactly proportional to the theoretical stresses, such a determination might easily be made. But in every girder material is introduced, according to the judgment of the Engineer, to cover joints and for stiffening, whilst the sectional area of the booms towards the ends and of the bracing or web towards the centre, cannot be reduced in practice to the dimensions which theory prescribes. The amount of this arbitrary excess of material far exceeds the difference in weight due to the form of the truss, and is not susceptible of any very exact estimation. Of the three forms of parallel braced girder, probably one is not much to be preferred to the other on the ground of economy of weight. Theoretically the lattice girder is the lightest and the Whipple-Murphy the heaviest of the three forms. Practically, making allowance for the excess of material above that theoretically necessary, the order is exactly inverted, and the Whipple-Murphy is probably by a small percentage lighter than the other forms.*

But if braced girders be compared with plate girders, a decided economy in the weight of the web may be fairly made out. Mr. Barton, the designer of the Boyne Bridge, has estimated the relative weight of a lattice, Warren, and plate web to be as follows:—

| | Material in Web. |
|---|---|
| Lattice... ... ... ... ... ... ... | 100 |
| Warren ... ... ... ... ... ... | 109 |
| Plate ... ... ... ... ... ... ... | 149 |

These proportions will be found to be confirmed by an examination of the ratio of the weight of the web to the total weight of the bridge in actual bridges. It must be borne in mind, however, in comparing the following figures, that the weight of the web increases while the weight of the booms decreases, when the ratio of span to depth decreases.

* Mr. Reilly makes the difference 10 per cent. in favour of the Whipple-Murphy truss, but in most cases the difference will not nearly reach that amount.

| | Ratio of span to depth. | Percentage of total weight. Top. | Bottom. | Web. | End pillars, &c. |
|---|---|---|---|---|---|
| PLATE WEBS. | | | | | |
| Britannia... ... | 15·5 | 30·6 | 31·6 | 37·8 | — |
| Conway ... ... | 15 | 33·2 | 33·3 | 33·5 | — |
| Victoria ... ... | 13 | 31·4 | 33·8 | 34·8 | — |
| BRACED WEBS. | | | | | |
| Boyne ... ... | 11 | 32·0 | 33·0 | 25·0 | 10·0 |
| Crumlin ... ... | 10 | 30·9 | 26·7 | 32·2 | 10·2 |

67. In defending the course he had adopted in employing tubular girders for the Victoria Bridge, Mr. Stephenson found it necessary to discuss the whole question of the relative advantages of different types of girder. Mr. Stephenson is the most distinguished advocate of the plate web, and his chief arguments are:* (1) That the booms being of the same weight, whatever the type of girder, the only advantage of one form over another must be a fraction of the weight of the web only. (2) That on the other hand, the continuous web does, in fact, assist the booms in resisting the horizontal forces, and therefore adds to the strength of the girder. (3) That the continuous web is more rigid than the braced girder.

If it were true that the economy of braced girders is confined to a fractional saving in the weight of the web, many Engineers, admitting the more simple and homogeneous construction of the plate web, would consider its superiority established. But a saving of weight in any part implies that the dead load to be carried is lessened, and therefore involves a cumulative saving in the weight of every part of the girder. In the next place, it is often possible to employ in the webs of braced girders a higher quality of iron than the plate iron used in continuous web girders, and this again results in a lightening of the structure. It has already been pointed out that the braced girder is not necessarily less rigid than the plate girder.

68. *Advantage of braced girders due to depth.* The most important source of economy in the braced girder remains to

* Report to the Directors of Grand Trunk Railway of Canada, 1855, and article "Iron Bridges," in "Encyclopædia Britannica."

be noticed, viz.: the greater depth which may be given to it. The depth of plate web girders is practically limited to from 1-12th to 1-15th of the span; but in braced girders a greater depth is constantly and successfully adopted.

It is true that, in this country, the influence of authority has tended to cause Engineers to look to the original tubular bridges, as establishing the proportions for all forms of girders. But even here, braced girders with a depth of 1-11th of the span are common; and in America, where the practice in this respect has been bolder and more original, a proportion of depth of 1-8th the span in large bridges and 1-7th in small bridges is a usual allowance.* The stress on the booms, and, consequently, their sectional area, is in the inverse ratio of the depth, and the saving of weight in the booms much more than compensates for the increased weight of the web.

The formula (§ 40) will not give accurately, with the same constant, the weights of girders of varying depth. But for a moderate variation in the depth it is probably near the truth. The three curves in Plate II. for the same value of $C$, show approximately how large is the influence of the depth on the weight.

69. *Ratio of depth of girder and width of boom to span in actual bridges.* The following table contains data of the proportions of existing bridges of all the types hitherto discussed. It should be remembered that in many of these bridges there is bracing between the top booms of two girders:—

| | Ratio of clear span. | |
|---|---|---|
| | To effective depth. | To width of top boom. |
| Britannia,† tubular ... ... ... | 16·7 | 31 |
| Conway, ,, ... ... ... | 16·9 | 27 |
| Aire Ferry ,, ... ... ... | 11·25 | 19 |
| Torksey, tubular girder ... ... | 13·65 | 52 |
| Penrith, ,, ... ... | 16 | 62 |
| Suir, ,, ... ... | 14 | 50 |
| Salt Water, ,, ... ... | 14·7 | 57 & 43 |

* Mr. Z. Colburn, in Proc. of Inst. of Civil Engineers, vol. XXII., p. 528.

† At centre of span.

| | Ratio of clear span. To effective depth. | To width of top boom. |
|---|---|---|
| Staines, plate girder ... ... | 12·6 | 44 & 35 |
| Cannon Street, ,, ... ... | 16 | 62 |
| Boyne, lattice ... ... ... | 11·7 | 88 |
| Charing Cross, ,, ... ... ... | 12 | 38 |
| Passau, ,, ... ... ... | 11 | — |
| L'Orient, ,, ... ... .. | 10 & 8 | 80 & 65 |
| Crumlin, Warren ... ... .. | 10 | 200 |
| Newark Dyke, ,, ... ... ... | 15 | 160 |
| Taptee, ,, ... ... ... | 8·6 | 42 |
| Connecticut, Whipple-Murphy... | 10 & 11 | 77 & 65 |
| Lough Ken, bowstring ... ... | 7·4 | 65 |
| Saltash, bowstring suspension ... | 8 | — |
| Maintz, ,, ... | 6 | — |
| Chepstow, trapezoidal truss ... | 7 | — |

## Determination of Stresses in Braced Girders.

70. In the determination of the stresses on braced girders, certain arbitrary assumptions are made to facilitate calculation. (1) The girder is assumed to be a completely jointed structure, no allowance being made for any additional strength due to the continuity of the members (as for instance, the booms) at the joints. In order that a jointed structure may be perfectly stable under variable loads, every bar must form a side of one or more triangles, the triangle being the only figure incapable of alteration of form, except by the variation of the length of its sides. (2) The girder is assumed to be loaded at the joints only. The live load and weight of platform is generally applied to the girder by cross-beams attached at or near to the joints of the top or bottom boom. That part of the load is therefore actually applied at the joints of one boom, in the proportion of the load corresponding to the distance between two consecutive cross-beams, at each joint at which a cross-beam is attached. In small girders the weight of the girder itself may be supposed to be applied, in the same proportion and to the same joints. But when the weight of the main girder is large, it is more accurate to assume that half its weight is applied at the joints of

the top boom, and the other half at the joints of the bottom

Suppose a girder of span $l$, the booms of which are divided into $n$ bays, so that $l \div n$ is the distance between two consecutive joints, in either boom. Let $w_1$, $w_2$, $w_3$, be the live load, external dead load (weight of platform, &c.,) and weight of girder itself, per foot run of span. Then for the boom on which the cross-girders rest, the load at each joint will be:—(*a*) If there is a cross-girder at every joint: $(w_1 + w_2 + \frac{1}{2} w_3) l \div n$. (*b*) If there is a cross-girder at every second joint: $(2 w_1 + 2 w_2 + \frac{1}{2} w_3) l \div n$ at the joints which carry the cross-girders, and $\frac{1}{2} w_3 l \div n$ at the intermediate joints. (*c*) If there is a cross-girder every third joint: $(3 w_1 + 3 w_2 + \frac{1}{2} w_3) l \div n$ at the joints which carry the cross-girders, and $\frac{1}{2} w_3 l \div n$ at the intermediate joints, and so on for other cases. The load at the joints of the boom which does not carry cross-girders will be $\frac{1}{2} w_3 l \div n$.

In some lattice bridges (the Charing Cross Bridge for example) a vertical suspension rod connects the corresponding joints of the top and bottom boom. The effect of this is to transfer part of the load resting at the bottom joint to the joint of the top boom, but the exact determination of the distribution of the load becomes a very complex problem, because not only are the bracing bars altered in length, but so also are the booms. An approximate calculation will, however, show that the proportional elongation of the vertical suspension rod will be about the same as that of the boom, so that the intensity of the stress on it will be about the same. Let the load on the joint be $W$, the stress on gross area of top boom 4 tons per square inch. Then, if the net area of the suspension rod is 0.1 $W$ square inches, the joints of top and bottom boom may be considered to be loaded equally.

71. (3) A third convenient assumption, which introduces only a very small error, and that on the safe side, is that in estimating the maximum shearing force, the girder is to be supposed loaded by the live load *per saltum*, at the joints where the cross-girders are attached. Practically, no joint can be fully loaded by the live load unless the joint in advance

of it is partially loaded, but it is more convenient to assume all the joints on one side of any section to be fully loaded, and all those on the other to carry the dead load only.

72. *Method of sections.* At any section of a vertically-loaded girder three equations may be found between the stresses on bars cut by the section and the external forces on either side of it:—

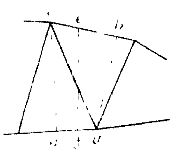
The sum of the vertical components of the stresses on the bars must be equal and opposite to the shearing force.

The sum of the horizontal components must be zero.

The sum of the moments of the stresses about any point in the section must be equal to the bending moment.

When sections can be taken cutting alternately two and three bars, a simple artifice enables us to ascertain the stresses directly. Let Fig. 54 be part of a braced structure. Let $M_1$ $M_2$ be the bending moments at $a$ and $c$, and $F$ the shearing force at the vertical section $ef$. Of the stresses on the bars cut by a section through $a$, perpendicular to $cb$, that at $b$ only has a moment about $a$; hence stress at $b = S_1 = M_1 \div ab$. Similarly, stress at $d = S_2 = M_2 \div cd$. Suppose these stresses ascertained, and their vertical components calculated by multiplying the stresses by the sines of the inclination of the bars to the horizon. Those vertical components, together with the vertical component of the stress on $ac$, must be equal and opposite to the shearing force $F$. Or putting $\alpha$, $\beta$, $\gamma$ for the inclinations of the bars, the stress on $ac$

FIG. 54.



$$= S_3 = (F - S_1 \sin. \alpha - S_2 \sin. \beta) \operatorname{cosec} \gamma.$$

If either of the vertical components of the stresses on the booms acts in the opposite direction to the shearing force, it will have to be added to instead of subtracted from $F$.*

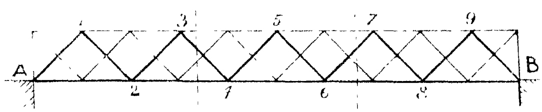
* This method was first described, I believe, by Prof. Rankine, as a systematic method for ascertaining the stresses in braced structures. It is the principle at the basis of all methods not depending on the successive resolution of the forces at the joints.

In girders of uniform depth, the stresses on the booms have no vertical components. Then $S_1 = M_1 \div h$; $S_2 = M_2 \div h$; $S_3 = F \operatorname{cosec} \gamma = F\,l \div h$; where $h$ is the effective depth of the girder, and $l$ is the length of a bracing bar.

If the girder has bracing arranged in several systems of triangulation, each system must be considered separately. The stress on the booms may then be found by adding up the stresses at any point due to each separate system of triangulation.

73. *Mr. Latham's method.* Let $AB$ be a braced girder, and let it be required to ascertain the shear on any bar 7 6.

FIG. 55.




The loads carried directly at the abutments may be neglected, and any stress on the bar 7 6 must be due to loads at 1, 2, 3, 4, 5, 6, 7, 8, 9. Let $W_1$ be the dead load on each top joint; $W_2$ the dead load on each bottom joint; $W_3$ the live load, which we will suppose to rest at each bottom joint. The maximum stress on 7 6 will occur when the live load covers the joints 2, 4, 6 only. Since there are ten bays in the span, and 2 is two bays from $A$, the reaction at $B$ due to a load at 2 is 2-10ths the load; that due to a load at 3 is 3-10ths the load, and so on. Hence, summing, the total reaction at $B$ due to all the loads at the joints of the triangulation under consideration is,

$$= \frac{W_1}{10}(1+3+5+7+9) + \frac{W_2}{10}(2+4+6+8) + \frac{W_3}{10}(2+4+6)$$

and the shear on 7 6, which is the reaction at $B$, less the loads between $B$ and a section through 7 6 is,

$$= \frac{W_1}{10}(1+3+5+7+9) + \frac{W_2}{10}(2+4+6+8) + \frac{W_3}{10}(2+4+6)$$
$$- 2\,W_1 - W_2$$
$$= \frac{W_1}{10}(1+3+5-1-3) + \frac{W_2}{10}(2+4+6-2) + \frac{W_3}{10}(2+4+6)$$

On examining this expression, it is easily seen to consist of the load at a joint divided by the number of bays and multiplied by an arithmetical series. Each of these series is the sum of the distances of the joints of the boom, belonging to the triangulation considered, on the left of the section, less the sum of those on the right, the unit being the length of a bay. Hence the shear on any bar of a braced girder, however complicated, can be written down in this form, directly, by the mere inspection of the drawing of a girder, and the calculation of its value is then a simple matter of arithmetic. The common difference in each of the arithmetical series is the number of bays between two consecutive joints of a single triangulation. The stress on each bar is simply the shear multiplied by its length, and divided by the depth of the girder, or otherwise the product of the shear and the cosecant of the inclination of the bar to the horizon. The stress on 3 4, when the rolling load comes on the bridge at *B*, will be the same as that on 6 7 when, as above, the rolling load comes on at *A*; and so on for other bars.

For the stress on the booms arithmetical series may also be formed, but not quite so conveniently. The safer process is to re-calculate the stresses on the bars for the case in which the live load extends over the whole bridge. The expressions will be the same as before, writing $W_2 + W_3$ for $W_2$, and omitting the term containing $W_3$. Thus for bar 6 7 the shear will be

$$F = \frac{W_1}{10}(1 + 3 + 5 - 1 - 3) + \frac{W_2 + W_3}{10}(2 + 4 + 6 - 2)$$

The stress on the bar 6 7 is then

$$F \text{ cosec. } \theta,$$

if $\theta$ is the inclination of the bar to the horizon; and the horizontal component of the stress on the bar will be,

$$F \text{ cosec. } \theta . \cos. \theta = F \cot. \theta.$$

Suppose the horizontal components of the stresses on all the bars in one half of the beam thus calculated. Then the stress on the boom between *B* and 8, due to loads at numbered joints, is the horizontal component of the stress on *B* 9. That on 9 7 is the sum of the horizontal components of the stresses

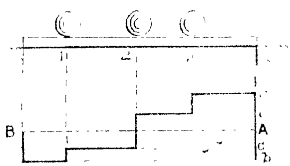
on $B$ 9 and 9 8. That on 8 6 is the sum of the horizontal components of the stresses on $B$ 9, 9 8, and 8 7, and so on.

If the number of bracing bars is very numerous, the stress on the booms calculated as for a plate girder is quite accurate enough for practical purposes.

74. *Graphic methods of obtaining the stress in braced girders.* The graphic methods already alluded to furnish in many cases the readiest means of ascertaining the stress in braced girders, but from the peculiarity in the loading at isolated and equi-distant points, it will be necessary to point out how the curves of shearing stress and bending moment can most readily be drawn.

75. *Graphic representation of shearing stress for a fixed load.* Take $A\ B$ to represent the span, and at right angles to $A\ B$ draw lines corresponding to the positions of the loads. On any scale of tons, take $A\ a$ equal to the reaction at $A$, and $A\ b$ equal to the reaction at $B$. Beginning at $a$, mark off $a\ c$, $c\ d$, $d\ b$, equal to the loads in the order of their distance from $A$. The sum of the loads is of course equal to the sum of the reactions.

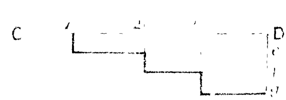
FIG. 56.




Through the points thus marked off, draw lines parallel to $A\ B$, to the vertical lines corresponding to each load. Thus the line from $a$ finishes at the first load. That from $c$ at the second, and so on. The ordinates of the stepped figure so obtained measured to $A\ B$ represent the shearing force at any point of the beam.

76. *Representation of maxima of shearing force during the passage of a travelling load.* Take as before $D\ C$ to represent the span $l$, and draw vertical lines corresponding to the points successively loaded by the rolling load. Let $W_1$, $W_2$, $W_3$, be the magnitudes of the loads at 1, 2, and 3; and $x_1$, $x_2$, $x_3$, the distances of those points from $C$. Suppose the train to come on the bridge at $C$. Take

FIG 57.



$De = W_1 x_1 \div l$; $ef = W_2 x_2 \div l$; $fg = W_3 x_3 \div l$. Through $e$ draw a line parallel to $C\,D$ to the vertical at 1; through $f$ a line to the vertical at 2; through $g$ a line to the vertical at 3. Then the ordinates of the stepped figure so obtained measured to $D\,E$ will represent the maximum shearing force at any section during the passage of a live load. In Fig. 56 the ordinates are positive upwards, considering the shearing force on the side of a section towards $A$. In Fig. 57 the ordinates are positive downwards, considering the side of a section towards $D$. They require to be so drawn when figures for dead and live load are to be combined, as in § 16.

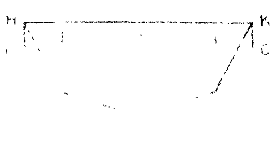
77. *Form of curve of bending moment.* For concentrated loads the curve of bending moment is a polygon, which in the cases ordinarily occurring in braced girders can be readily obtained from the parabolic curve which represents the distribution of bending moment due to a uniform load.

*Case* 1. Equal and equi-distant loads; distances of extreme loads from abutments equal to distance between two consecutive loads. In this case it may easily be verified that the bending moment at each load is the same as that of an equivalent distributed load. So that if the parabola representing the bending moment of a distributed load of $w = W \div d$ tons, per foot run, ($W$ being the load at a joint in tons, and $d$ the distance between two joints in feet,) be drawn (§ 21), the actual distribution of bending moment will be represented by the polygon inscribed in that parabola, and having its vertices, vertically under the loaded joints (Fig. 58).

58.


*Case* 2. Equal and equi-distant loads; distances of extreme loads from abutments equal to half the distance between two consecutive loads. Draw the curve for a uniform load $w$ as above. Take $F\,H = \frac{w\,d^2}{8}$; draw $H\,K$ parallel to $F\,G$, and inscribe the polygon with vertices under the

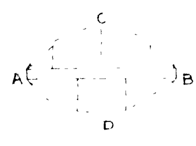
FIG. 59.



loaded joints. Then the ordinates of the polygon measured to $H\ K$ will be the values of the bending moments.

*Case* 3. Loads on one boom corresponding to Case 1, and on the other boom to Case 2. Draw the polygon $A\ C\ B$ to represent the bending moments due to the latter, and on the other side of $A\ B$ draw $A\ D\ B$ to represent the bending moments due to the former. The bending moment at any point is the whole breadth of the figure so obtained.

FIG. 60.




78. *Representation of total stress on boom.* The total stress on boom may be represented by polygons derived from the curve of total stress for girders of uniform depth (§ 25), in precisely the same way as the polygons of bending moment. Only substituting $w\ d^2 \div 8\ h$, for $w\ d^2 \div 8$ in Cases 2 and 3. The polygon so obtained is not, however, the true representation of the distribution of stress. On any segment of the boom the stress is uniform from joint to joint, and the magnitude of the stress will be the ordinate of the polygon obtained as above, corresponding to the joint of the other boom opposite that segment. Thus in Plate V., Fig. 4, the parabolic curve of total stress for a uniformly distributed load has first been drawn. In that has been inscribed a polygon with vertices corresponding to the loaded joints. Then $q$ is the point corresponding to joint 10 (Fig. 1), and $s\ q$ is the value of the total stress on 11-9, the segment opposite joint 10; $r$ is the point of the polygon corresponding to joint 9, and $r\ t$ the value of the stress on 10-8. Hence the actual distribution of stress is obtained by drawing lines $m\ n$, $o\ p$, parallel to $A\ B$, and of length corresponding to the segments of the booms. The stepped figure in full lines so obtained represents the distribution of stress on the bottom boom; that in dotted lines the stress on the top boom. The figures so obtained may be used in designing the longitudinal section of the boom, as in § 47.

The following examples of the application of graphic methods may assist the student in applying the principles already indicated.

79. *Warren girder.* Joints of one boom only supposed loaded; extreme loads distant one bay from abutments. Plate V. In this, the simplest case which can be taken, the girder is supposed to be 50 feet span, and each of the joints 3, 5, 7, 9, is permanently loaded with 10 tons, and carries 10 tons live load during the passage of a train. The equivalent uniformly distributed dead load is 1 ton per foot run, and live load also 1 ton per foot run. (1) *Maximum stress on bracing.* Take $AB$ (Fig. 2) to represent the span; $AC$ = half the total distributed dead load (25 tons); $AD$ = half the total distributed live load (25 tons); and draw the diagram of maximum shearing stress as in §21. Then the maximum shear in each bay will be the ordinate of the figure corresponding to the centre of the bay. Thus, $ab$ is the shear on the bars 11-10, 10-9; $cd$ on the bars 9-8, 8-7. To find the actual stress on the bars, draw $gh$ through $a$ parallel to $AB$; then $bh$ from $b$ parallel to 11-10 represents on the scale of tons the stress on that bar; $bg$ parallel to 10-9 the stress on that bar, and so on. Having ascertained the stresses on the bars up to the point where the shearing force changes sign, it is only necessary to write the same values against the corresponding bars beginning at the other abutment, for the stresses when the train comes on the bridge at $A$ instead of at $B$. Some of the bars at the centre may thus have two stresses written against them, one compressive, the other tensile. These bars are those to which counterbracing must be applied.

(2) *Stress on booms* obtained from diagram of shearing force. For the stress on the booms the live load is to be assumed to extend to the whole bridge. Fig. 3 is the corresponding diagram of shearing force for the equivalent uniformly distributed load. $AD = AC = 25$ tons as before; $ab$ opposite the centre of the first bay represents, as before, the shear on the bars of that bay; $bg$, $bh$, drawn parallel to the bars, represent the actual stresses on bars 10-9, 10-11; and $ga$, $ah$, represent the horizontal components of those stresses. Similarly $kc$, $cl$, are the horizontal components of the stresses on 8-7, 8-9. Now the stress on the segment of the boom 11-9 is equal to the horizontal component of the stress on bar 11-10;

that on 10-8 to the horizontal components of the stresses on 11-10, 10-9; that on 9-7 to the horizontal components of the stresses on 10-11, 10-9, and 9-8, and so on, *taking* in each case all the bars between the segment under consideration and the abutment.

(3) *Stress on booms* from curve of total stress. In Fig. 4 the curve of total stress for the equivalent distributed load has been drawn as in §25, and the polygon representing the stress for the actual concentrated loads has been inscribed in it, with its vertices opposite the loaded joints 3, 5, 7, 9. Then the stress on each segment of either boom is the value of the ordinate of the polygon, corresponding to that joint of the other boom which is opposite the segment considered. Thus the full lines opposite the joints 10, 8, 6, represent the stresses on the segments at the bottom boom, and the dotted lines opposite the joints 9, 7, represent the stress on the segments of the top boom. The stresses are identical with those obtained by the previous process.

80. *Lattice girder;* top and bottom joints differently loaded. Plate VI. In this case the girder assumed for illustration is 50 feet span and 10 feet deep, with four systems of triangles in the bracing. We shall consider only the stresses due to the loads at joints 1, 3, 5, 7, 9, of one of these systems. The stresses on the bars of the other systems and on the booms due to the loads at the other joints will be obtained in a precisely similar manner. The dead load at each top joint (due to half weight of girder) is assumed at 0·5 tons; that at each bottom joint (due to half weight of girder and weight of platform) at 3 tons; and live load at each bottom joint 5 tons. This corresponds with a distributed load per foot run of 0·2 tons for girder, 0·5 tons for platform, and 1 ton for rolling load.

*Stress on bracing bars.* Draw first the diagram of shearing force of dead load as in §75. Take $AC$ the reaction at left abutment, $AD$ that at right abutment, and mark off from $C$, $Cd$, $dc$, $cb$, $ba$, $aD$, equal to the dead load at joints 9, 7, 5, 3, 1. From $C$ draw a horizontal line to the vertical from 9; from $d$ a horizontal between the verticals from 9, 7; and so on. Next draw the diagram of maximum shearing force due to

the travelling load, for which joints 3, 7, only require consideration. The reaction at 10 due to 5 tons at 3 is $4 \times 3 \div 10 = 1\cdot5$; that due to 5 tons at 7 is $5 \times 7 \div 10 = 3\cdot5$. Set off $A e = 1\cdot5$, $ef = 3\cdot5$, and draw horizontals from $f$ to the vertical from 7; from $e$ between the verticals from 7, 3. The figure of shearing force is then complete. The breadths of the diagram are positive from $A$ to $K$ and negative from $K$ to $B$ (compare § 16). The actual stresses on the bars are given by the lengths of lines drawn parallel to the bars between the lines representing the shearing force in the half bay to which they belong; thus, 10-9 is the stress on bar 10-9; 9-7 on bar 9-7; and so on. The stresses having been written against the bars up to the point where the shearing force changes sign, the same values must be written against corresponding bars beginning at the right abutment, for the stresses in the case when the rolling load comes on the bridge at $A$ instead of at $B$.

*Stresses on booms* due to bars in triangulation considered. Next draw the diagram of shearing force, supposing the rolling load to cover the whole bridge (Fig. 3). The diagram for the dead load will be the same as in Fig. 2, but for the live load take $A G$ = reaction at $A$ of live loads on 7, 3, and $BF$ = reaction at $B$. In this case $A G = BF = 5$ tons. Draw horizontal lines from $G$, $F$, to the verticals from 7, 3, the diagram of shearing force is then complete for the system of triangulation considered. As before, $Gh$ will be the stress on 10-9, and the horizontal component of that stress will be $Ch$. Therefore $Ch$ is the stress brought on the top boom at 9 by the bar 10-9. The horizontal components of the stresses on all the other bars must be similarly found. Then the total stress on the boom at any point is the sum of the horizontal components of the stresses on all the bars between that point and the nearest abutment. It is, of course, only necessary to determine the stresses thus on half the boom, and to write off the stresses found for each bar in the corresponding bay of the other half.

*Stress on booms.* Approximate method. Draw the curve of total stress (§ 25) for a uniform load (Fig. 4). Then the stress

in each bay is approximately the ordinate corresponding to the centre of the bay. In this case the equivalent uniform load is $0{\cdot}5 + 0{\cdot}2 + 1{\cdot}0 = 1{\cdot}7$ tons per foot. The total uniform load is $1{\cdot}7 \times 50 = 85$ tons, and one-fourth of that is $21\frac{1}{4}$ tons. Set off $AB$ = span; $CD$ = depth of girder; $CE = 21\frac{1}{4}$ tons. Join $DE$ and draw from $B$, $BF$ parallel to it. Bisect $BF$ in $G$ and draw $GH$ perpendicular, cutting $FC$ in $H$. From centre $H$ with radius $HF$ draw the circular arc $AFB$, representing the curve of stress. Then the ordinates opposite the centre of the bays of the booms, measured from $AB$ to the curve, will be approximately the value of the total stress on the booms. In the present instance, near the centre of the girder, the values so found are almost exact for the bottom boom and about 1-17th too great for the top boom. Towards the ends the proportional error is greater, but of less consequence, because the section of the boom at the ends is always much larger than is theoretically necessary.

81. *Bowstring girder.* To draw the diagram of stress on arched boom. Plate VII. The girder is supposed to be 50 feet span with single triangulation; each bottom joint loaded with 10 tons, equivalent to $1{\cdot}4$ tons per foot run. Draw the curve of moments (§ 77). The stress on any segment of the arched boom is the bending moment at the opposite joint of bottom boom, divided by the perpendicular from that joint on the segment. Let $C$ be the centre from which the curve of the boom is struck, draw a radius through 5, then the stress on 8-9 is the bending moment $ef$, divided by the distance $c$-5. On any line $hk$ set off $gh$ = the moment $ef$; $gk = c5$; draw perpendiculars $kl$, $hm$; suppose there are $n$ units of the scale of feet in a unit on the scale of foot tons to which the curve of moments is drawn; then $kl$ is to be taken $= n$ tons on the scale of tons; join $lg$ and produce to cut $hm$; then $hm$ is the stress on 8-9.

To draw the curve of stress, take $HK$ a tangent to the boom; take $CO = 2\ CD$ ($O$ is necessarily displaced on the plate); from $O$ draw lines through $A$ and $B$, cutting the tangent in $H$, $K$, then $H K$ is the development of the boom, and lines from $O$ through the joints will give their position

on the development. At the position of the joints raise perpendiculars, then the stress being uniform in each of the bays, take $a\,d = h\,m$, and draw a line parallel to the tangent for the stress in the bay corresponding to $8-9$, and so on for the other bays. For the extreme bay $A\ 7$, take $p\,q =$ reaction due to loads at $A$, and draw $p\,r$, $q\,r$, parallel to $A\ 6$, $A\ 7$; then $q\,r$ is the stress on $A\ 7$, and set-off at $H$ will give the height of the last division of the curve of stress. The diagram of stress so obtained may be used in designing the longitudinal section of the boom, precisely as the parabolic curve in § 47.

## QUALITY OF IRON, TESTS, &c.

82. Two or three points connected with the practical construction of bridges remain for consideration, of which the most important is the quality of iron to be used and the tests to which the iron may be subjected. Two tests of the iron used by manufacturers are employed; in some cases the whole of the iron employed is tested to some limited stress, and inferences are drawn from the elongation and permanent set observed; in other cases certain portions of the iron to be used are selected and tested to the breaking point. The information derived from the first method is very imperfect at best, and its chief use is to detect any extreme instances of bad material or imperfect manufacture in the links of suspension bridges, which being all of similar dimensions should give the same indications under proof; so that if in any one bar a variation is found to exist, some imperfection of manufacture may fairly be inferred. The information derived from the ultimate test of a portion of the iron to be used is most definite and valuable, and though it does not serve to detect individual instances of bad material, it affords an absolute criterion of the average strength and ductility of the iron, and the degree of uniformity attained in its manufacture. It is quite as important generally, in rivetted structures, that the iron should be of uniform quality as that it should have a high average strength.

The ordinary plate iron used for bridges cannot be depended

on to carry more than 20 tons tensile stress per square inch, and there is known to be some difficulty in obtaining it at a reasonable rate, under a guarantee that the breaking weight shall not fall short of 22 tons. For bar iron 24 or 25 tons may be exacted, and the links of suspension bridges, in the manufacture of which more care is expended, have sometimes an ultimate strength of 28 tons. The one irremediable quality in iron intended for a rivetted structure is brittleness or want of ductility. The best iron is that which is tough as well as strong. Brittleness is accompanied by a small elongation under strain, and generally by a bright silvery crystalline or granular fracture.

The proof stress for iron not intended to be broken varies from 1-3rd to 1-2nd of the ultimate strength; plate iron being tested to about 7 tons, and bar iron often to 12 tons. The links of the Chelsea suspension bridge were tested to 13½ tons, under the condition that any link should be rejected in which the extension under that stress amounted to 1-40th of an inch in 10 feet or 1-6480th of the length per ton. The actual extension observed only amounted to from 1-14000th to 1-15000th. It is often exacted that all bars should be rejected in which a permanent set is perceptible under the proof stress, a provision which, except as a precaution against want of uniformity, does not seem desirable. There must be a permanent set in any bar not previously tested, under a proof stress, perceptible if the means of measurement are sufficiently delicate; and on the other hand any manufacturer who wished to evade the precaution need only strain the bars to about the proof stress before delivery, to ensure their passing the test.

In fixing the minimum limit of ultimate strength, it must be remembered that plate iron, such as is used for bridges, is from 1-6th to 1-7th stronger in the direction in which it has been rolled than across it. It is usual to specify that the elongation at some fixed stress should not exceed a defined amount. But it should be remembered that a large elongation is not necessarily an evil. If due to the toughness and great range of elasticity of the material, it is a merit and not

a defect, and the greater the elongation the less will be the inequality in the distribution of stress in the structure, arising from small but unavoidable inaccuracies of workmanship. On the other hand, a large elongation at a given stress may be due to the low resistance of the material, and to the fact that the given stress is near the breaking point. A small elongation may be due to the hard crystalline nature of the material, or to the breaking stress being considerably greater than that at which the elongation is measured. To secure uniformity in the quality it would seem to be quite as necessary to specify that the elongation should not fall short of a given limit, as that it should not exceed some other given limit. The ultimate elongation is only 2-3rds to 1-2nd as great across the fibre as in the direction of the fibre. The average ultimate extension per unit of length decreases with the length of specimen tested; and varies apparently in the same material with the thickness to which the plates have been rolled, thinner plates being generally more rigid and less ductile than thick plates, at least for a range of from $\frac{1}{4}$ to 1 inch in thickness.

83. *Economical span.* If a river has to be crossed in a series of spans, it becomes a question of interest to determine with what span the cost will be least. If the weight of the main girders is assumed to vary as the square of the span, which is approximately true, and the cost of the piers to vary as their number only, independently of the span, which is also approximately true, then it can be shown that the cost of the bridge will be a minimum when the spans are so arranged that the cost of the main girders for one span is equal to the cost of a pier; a simple rule, due it is believed to Mr. Liddell, the Engineer of the Crumlin Viaduct. The formula in § 40 will enable a nearer approximation to be made. Let $L$ be the space to be bridged; $n$ = number of spans; $l$ = one span, so that $n = L \div l$ very nearly; $P$ = cost of one pier; $Q$ = cost of ironwork of main girders, erected, per ton. Then the other notation remaining as in § 40, and neglecting the cost of abutments, platform and permanent way, which will not vary with the span adopted :—

Cost of piers $= (n - 1) P$

Cost of main girders $= \dfrac{(w_1 + w_2) L^2 r Q}{C s n - L r}$

Whole cost of part of bridge variable with span is—

$$y = (n - 1) P + \frac{(w_1 + w_2) L^2 r Q}{C s n - L r}$$

For the value of $n$ which makes $y$ a minimum—

$$\frac{dy}{dn} = P - \frac{(w_1 + w_2) L^2 r Q C s}{(C s n - L r)^2} = 0$$

$$\therefore n^2 - \frac{2 n L r}{C s} + \frac{L^2 r^2}{C^2 s^2} - \frac{(w_1 + w_2) L^2 r Q}{C s P} = 0$$

$$\therefore n = \frac{L}{C s}\left(r \pm \sqrt{\frac{(w_1 + w_2) r Q C s}{P}}\right)$$

The upper sign giving the value of $n$ for the minimum. Hence the most economic span is—

$$l = C s \div \left(r + \sqrt{\frac{(w_1 + w_2) r Q C s}{P}}\right)$$

Thus taking $C = 1500$; $s = 4$; $r = 10$; $w_1 + w_2 = 1.5$ for a single-line bridge, and $= 3.0$ for a double-line bridge; $Q =$ £24;

| Cost of one Pier. | Most economic span in feet. Single Line. | Double Line. |
|---|---|---|
| £1,000 | 106 | 80 |
| 2,000 | 140 | 106 |
| 5,000 | 194 | 152 |
| 10,000 | 243 | 194 |
| 15,000 | 272 | 222 |

It is worthy of notice that since the economical span increases, as the ratio of depth to span decreases, there will be, in a series of spans, not only a saving of absolute weight in the superstructure, but also in the cost of the piers, from the adoption of a type permitting a greater depth in the girders. For girders continuous over the spans the value of $C$ may be taken 1-4th greater than for discontinuous girders.

84. *Camber and deflection with live load.* In similar beams similarly loaded the deflection is proportional directly to corresponding linear dimensions, for instance to the span. In girders similar in other respects, but differing in the ratio of depth to span, the deflection is directly as the square of the span and inversely as the depth; and in all flanged girders the deflection within the limits of proof stress is proportional to the mean stress per square inch on the booms. Hence if $\delta$ = the total deflection (exclusive of set) with the working load, in inches; $l$ = the span in feet; $h$ = the effective depth in feet; $s$ = the mean intensity of stress on booms;

$$\delta = \frac{l^2 s}{v h} = \frac{l r s}{v}$$

where $v$ is a constant. For determining the camber to be given to a girder, in order that with an ordinary load it may be horizontal, it is sufficiently accurate to assume that the set is proportional to the deflection, so that if $\delta_1$ be the camber required, $\delta_1 = lrs \div v_1$; where $v_1$ appears to range in different examples from 3,000 to 4,000. The actual camber usually given during construction is about 1-4th greater than this.

From § 40 we obtain for the ratio of live load to total load,

$$k = \frac{w_1}{w_1 + w_2 + w_3} = \frac{w_1 (C s - l r)}{(w_1 + w_2) C s}$$

so that if $\delta_2$ be the deflection due to the working load,

$$\delta_2 = k \delta = \frac{w_1 (C s - l r) l r}{(w_1 + w_2) C v}$$

an approximate formula, convenient because expressed in terms of quantities, known during the design of the girder. Putting $A$ for the gross area of two booms, as in § 35, it is easily shown that

$$\delta_2 = \frac{w_1 l^2 r^2}{4 A v}$$

which is the more convenient formula for deducing the deflection from the proof load. The value of the constant $v$ in some large bridges ranges from 4,000 to 6,000.

# LECTURE V.

## *ON RIVETTED AND OTHER JOINTS.*

In the present Lecture attention will be directed to a few of the more important principles which should regulate the design of the joints of wrought-iron structures, composed of bars and plates.

85. In the first plate-iron bridges, two forms of rivetted joint were used, founded on a theory of this kind: It was supposed that in the top boom and other parts in compression, the plates, being carefully butted against each other, would transmit the stress equally well whether a joint existed or not; and hence, that the entire office of the rivets and covering plates was to keep the plates in position and to bind them together, to act as a single mass, without themselves transmitting any part of the stress. The single rivetted butt joint was therefore used for the joints in compression, with a narrow covering strip and a single line of rivets on each side of the joint, pitched about 3 inches apart. The joints in tension were of a different kind. It was perceived that at such a joint the whole stress must be transmitted through the rivets and covering strip, and that the strength of the plate would be necessarily reduced, in the exact proportion of the quantity of metal punched out, in the line of fracture across the plate. Dr. Fairbairn, therefore, invented the form of rivetting since known as *chain rivetting* (Fig. 63), in which the rivets are at about 4 inches pitch across the plate, and the requisite shearing area is obtained by placing the rivets one behind another, in rows, with a long

covering plate of equal or slightly greater strength than the plates to be united. With the single rivetted joint, in tension, the strength is reduced, in consequence of the metal punched out, by 50 per cent., but in a well-designed chain joint the loss of strength should not exceed 15 per cent.

Experience has shown that it is not always safe to depend on the plates of the compression booms being so accurately butted against each other, that they will transmit the thrust direct. With a slight inaccuracy of workmanship, in shearing the plates or fitting them together, the thrust is thrown through the rivets into the covers. It has also been found that in addition to tearing across the line of fracture or shearing through the rivets, in either of which cases a joint gives way suddenly and altogether, a joint may have a hidden source of weakness, which, without causing its total destruction, gradually deteriorates it. This is deficiency of bearing surface between the rivet and the plate, resulting in a gradual crushing of the surfaces in contact, the enlargement of the rivet holes, and the loosening of the joint.*

86. *Resistance to shearing.* The safe limits of stress in tension and compression, for iron as ordinarily used in bridges, have already been given (§ 33). Experiment shows that the resistance to shearing varies simply as the area, and, with rivets closely fitting the rivet holes, is equal to or very little less than the resistance to tension. Hence, provided the stress were uniformly distributed to all the rivets of a joint, the shearing area through the rivets ought to be equal to the tearing area through the plates, or perhaps since the iron used for rivets is rather stronger than ordinary plate iron, the shearing area should be rather less than the tearing area. But with the large number of rivets in a bridge joint it is scarcely possible that the stress should be uniform on

* The bearing area of a rivet is the product of its diameter and the thickness of the plate on which it bears. It was clearly shown in Dr. Fairbairn's experiments on punching that, for equal pressures on the punch, nearly equal volumes were displaced, whether the punch were flat ended or hemispherical, and a similar law would probably hold with other forms. And hence the resistance to crushing will be as the cross-section, and not as the surface.

them all, and since the safety of the joint is measured by the maximum and not the mean stress, it is safer to take the resistance to shearing at from 1-10th to 1-5th less than the resistance to tension, in proportioning the rivets, so that the excess of area may make up for the inequality of stress.

87. *Safe limit of pressure on bearing area.* The safe limit of pressure on the bearing surface is much more difficult to determine. Mr. Latham, who first called attention to this element, fixed it at 5 tons per square inch, but apparently without much experimental basis for the assumption. When a short prism of wrought iron is subjected to compression, it takes a measurable permanent set with less than 10 tons per square inch, and in long columns Prof. Hodgkinson detected a permanent set with less than one ton. But in these cases the iron is entirely unsupported by neighbouring material.

When a wrought-iron plate is supported on a die block, having a hole $1\frac{1}{4}$ inches diameter, and pressed by a punch 0.87 inches diameter, the indentation of the plate by the punch becomes perceptible when the pressure reaches 11 tons per square inch, in plates $\frac{1}{4}$ or $\frac{1}{2}$-inch thick; and 28 tons in plates $\frac{3}{4}$ or 1-inch thick. The difficulty of observing the exact point at which indendation begins, preventing the determination of the exact relation between the indenting pressure and the thickness of the plates.* Now the metal in front of the punch, in these experiments, was supported by the resistance to shearing of the neighbouring parts, in a manner not essentially very dissimilar to that in which the metal in front of a rivet is supported by its adhesion to the metal between the rivet holes.

In Dr. Fairbairn's experiments on rivetted joints, there is a remarkable connexion, which has apparently escaped notice, between the bearing surface of the rivets and the strength of the metal between the rivet holes. All the joints, in those experiments, gave way by the tearing of the metal between the rivet holes, but the tension per square inch at which the plate gave way varied from 19 to 27 tons, being always

* Experiments by W. Fairbairn, Esq., F.R.S., for the Iron Plate Committee.

greater in joints of the same kind, when the bearing surface of the rivets was greater, as the following reduction of the results will show:—

*Strength of rivetted joints.*

| | Form of Joint. | Shearing area. sq. in. | Bearing area. sq. in. | Tearing area. sq. in. | Ultimate strength of net section of plate in tons, ⅌ sq. in. |
|---|---|---|---|---|---|
| 1 | Lap, single rivetted ... | ·40 | ·22 | ·44 | 18·7 |
| 2 | " " ... | ·60 | ·33 | ·44 | 19·9 |
| 3 | Lap, double rivetted ... | ·60 | ·33 | ·44 | 23·3 |
| 4 | " " ... | ·60 | ·33 | ·52 | 21·8 |
| 5 | " " ... | ·55 | ·41 | ·41 | 26·1 |
| 6 | " " ... | ·55 | ·41 | ·38 | 24·7 |
| 7 | Butt, double, 1 cover ... | ·55 | ·41 | ·44 | 24·1 |
| 8 | " single, 2 covers ... | ·33 | ·25 | ·44 | 23·9 |
| 9 | " double, 2 covers... | ·55 | ·41 | ·44 | 27·1 |

Comparing experiments 1 and 2; 4, 3, 5 and 6; and lastly, 8 and 9, the resistance of the iron increases uniformly with the bearing surface, and attains a maximum when the bearing surface is equal to the tearing area. This result is singularly in accordance with the results of some experiments by Sir C. Fox on the links of suspension bridges, which will be alluded to presently. Nor can it be explained by the frictional adhesion of the plates, which is known to be considerable, but which does not much affect the resistance of the plate to tearing at the first row of rivets.

It does not follow, because in joints which are to be fractured the bearing should be equal to the tearing area, that the same ratio should hold in joints on which the stress is within the limits of elasticity. The joints of the Crumlin Viaduct sensibly enlarged with a pressure on the bearing surface which probably reached 15 tons to the square inch; but there is every reason to believe that they would have been durable had the bolts been of twice the diameter and the pressure on the bearing surface reduced to 7½ tons per square inch, and that limit appears to be consistent with the experiments on indentation. Mr. Latham has mentioned that with a stress of 7 tons per square inch of bearing area left for

some days on a girder, some of the rivets of the joints will be found marked by the plates. It does not appear to the author that this is at all inconsistent with the perfect durability of the joint under that stress. For the minute yielding of one or two rivets will merely bring all the rest to a bearing, and tend to equalise the stress throughout the joint. Probably $7\frac{1}{2}$ tons may be taken for the limit in chain-rivetted joints, where from the distance of the rivets from the edge they are well supported, and Mr. Latham's limit of 5 tons in other cases, with perfect security.

88. In the following paragraphs $W$ is the total load or stress on a joint in tons; $s_1$, $s_2$, $s_3$ the safe limit of stress in tension, compression and shear, in tons per square inch; $s_4$ the limit of pressure on the bearing surface.

## Joints in Tension.

89. *Suspension link* (Plate IV., Fig. 2). The simplest joint which can occur is when a suspension link is supported by a bolt between two other plates and is subject to a longitudinal tension. Let $b$ be the breadth of the body of the bolt, $b_1$ the width of metal round the bolt, $t$ the thickness of the link, $d$ the diameter of the bolt, the dimensions being in inches. Then so far as shearing and tearing are concerned, the arcas would be sufficient, if

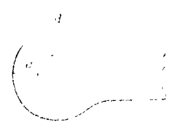
$$W = s_1\, b\, t = 2\, s_1\, b_1\, t = \tfrac{1}{2}\, \pi\, s_3\, d^2$$

provided that the stress on the sections was uniformly distributed.

But if the stress is not uniformly distributed, the area must be so much larger that the defined limit of stress is not exceeded at the point where the intensity of the stress is a

FIG. 61.

FIG. 62.



maximum. For example, suppose the bolt slightly smaller than the bolt hole, then the head of the link will become slightly elliptical under the action of the tension, and there will be a compression at *a* and *d* and a tension at *c* and *b* due to flexure, and additional to, and independent of, the direct stress on the sections at those points. Again, suppose the bolt originally to fit the hole accurately, but that, in consequence of the intensity of the pressure, the metal at *a* is crushed. Then a space will be left behind the bolt, and the hinder half of the head will become elliptical, the greatest flattening of the curve, and consequently the greatest stress, occurring at two points *e g* behind the vertical centre line. The link will give away by tearing from *e* and *g* outwards as soon as the intensity of the stress at *e* and *g* reaches the ultimate resistance, and in some cases long before the average direct stress on the section reaches that amount. This is the explanation of a remarkable and at first sight paradoxical result, obtained by Sir Charles Fox, in testing the links of a suspension bridge.* The links were originally 10¼ inches wide in the body, 16½ inches diameter of head, 1 inch thick, and bored out for a 4½-inch bolt. The iron was of a tenacity of 27 tons. When subjected to test the links gave way at the lines *ef*, *gh*, under a load of 180 tons. When the same links were bored out for a 6-inch bolt they supported 240 tons before fracture, or a greater load on a less section of metal. In the former case the metal at *a* was very much crushed, and the hole drawn into a pear-shape. In the latter case the intensity of the pressure on the bearing surface was insufficient to crush the metal, and flexure was almost entirely prevented by the fitting of the bolt. Sir Charles Fox was led by these experiments to recommend that the diameter of the bolt should not be less than 2-3rds of the width of the body of the link ($d = \frac{2}{3} b$), independently of its resistance to shearing; and that, as an additional precaution, the sum of the widths of the metal round the bolt should be 1-10th greater that the width of the body ($b_1 = {\cdot}55\, b$). The bearing surface of the bolt on the link is $dt$, and according to these experi-

* Proceedings of Royal Society, 1865.

ments the pressure on that surface should not exceed 40 tons per square inch when the link is on the point of breaking, which gives the safe working pressure on the bearing one-half greater than the limit of stress in tension.

90. *Rivetted plate tie bar.* The next simplest form of joint which can be used in bridge-work is that shown in Plate IV., Fig. 1, where a tie bar is lapped over a plate of the boom and rivetted to it. In this case the rivets shear in one plane only. If the rivets are properly arranged, the tearing area of the tie is diminished by one rivet only. Thus the tie may tear across the section $ab$; in which there is one rivet hole, or through the section $cd$, in which there are two rivet holes, but the stress on the latter section is less than on the former by the stress transmitted to the boom by the first rivet, and if the tie give way in the section $cd$ it must also at the same time shear the rivet in the section $ab$. Hence, if the diameter $d$ of the rivets is so arranged that

$$\tfrac{1}{4}\,\pi\, d^2 s_3 \text{ is not less than } dts_1$$

the section $cd$ will be as strong as $ab$. Hence, putting as before, $b$ = breadth, and $t$ = thickness of tie in inches; $d$ = diameter, and $n$ = number of rivets, we must have:—

| | | |
|---|---|---|
| Tearing area | ... | $(b-d)\,t = W \div s_1$ |
| Shearing area | ... | $\cdot 78\, nd^2 = W \div s_3$ |
| Bearing area | ... | $ndt = W \div s_4$ |

(Where the sign of equality is to be understood to mean equal to or greater than.) As the stress is very unequally distributed to the rivets, a low value should be taken for the shearing stress.

91. *Chain rivetting.* The common form of rivetting in the tension booms of wrought-iron bridges is one introduced by Dr. Fairbairn, which to great strength unites great facility of construction. In this system the rivets are placed one behind another in rows parallel with the length of the girder, and with 3, 4, or 5 rivets on each side of the joint. (Fig. 63.) When two plates are to be united there should always be a cover plate, of half the thickness of the plates to be united, on each side of the joint (Fig. 65). When several plates are

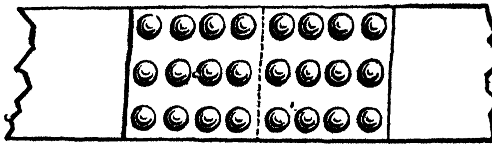


FIG. 63.

FIG. 64.

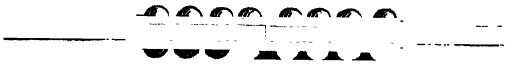


FIG. 65.

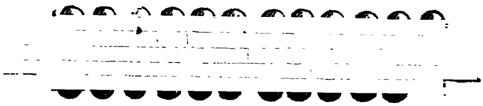


FIG. 66.

with a joint in one of them (Fig. 64), the cover plate is often put on one side only, and is then of the thickness of the plate in which the joint occurs. But in booms in which there are several thicknesses of plates, both weight and workmanship may be economised by the arrangement shown in Fig. 66, where the joints are brought within a distance equal to twice the longitudinal pitch of the rivetting, and the same cover plates answer for all the joints. It is clear, however, that in this case when a joint occurs in the top plate, the greater part of the tension in that plate will for a short distance be thrown into the top cover plate, and when a joint occurs in the bottom plate, into the bottom cover plate. It is only when the joint is midway between the cover plates that they share the stress equally. Hence the cover plates should each be nearly of the thickness of the plates to be united.

Let $m$ = number of rivets in one line parallel to the joint; $n$ = total number of rivets on one side of the joint; $d$ = diameter of rivets; $t$ = thickness of plates; and $b$ = breadth of joint in inches. Then we must have:—

Tearing area ... $(b - md)t = W \div s_1$ ...... (1)

Bearing area ... $ndt = W \div s_4$ ...... (2)

Shearing area

One cover $\cdot 78\, nd^2 = W \div s_3$ ...... (3)

Two covers $1 \cdot 56\, nd^2 = W \div s$ ...... (4)

Where there are three equations and four dimensions $m$, $n$, $b$, and $d$, to be determined in any given case. Of these $m$ is usually fixed arbitrarily by the condition that a certain proportion only of the plate is to be punched out (about 15 per cent. in many cases), or from structural considerations. From the other equations we get:—

One cover plate ... $d = s_4 t \div \cdot 78\, s$

$n = \cdot 78\, s_3\, W \div s_4^2\, t^2$

(or if $m$ is fixed) $n = (\cdot 78\, s_3 b - m s_4 t)\, s_1 \div s_4^2 t.$

And the same equations with 1·56 substituted for ·78 if there are two cover plates. Suppose $s_1 = 5$; $s_3 = 4\cdot 5$; $s_4 = 7\cdot 5$. Then

One cover plate ... $d = 2\cdot 14\, t$

$n = \cdot 062\, W \div t^2$

(or if $m$ is fixed) $n = (b - 2\cdot 13\, mt) \div 3\cdot 2\, t.$

Two cover plates ... $d = 1\cdot 07\, t$

$n = \cdot 124\, W \div t^2$

(or if $m$ is fixed) $n = (b - 1\cdot 06\, mt) \div 1\cdot 6\, t$

92. These are theoretically perfect joints, were there no restriction on the diameter of the rivets. In any other arrangement of sufficient strength, there will be an excess of either shearing or bearing area. Now in practice there is a very imperative restriction on the diameter of the rivets, arising out of the practical conditions of the punching and rivetting processes. It is possible, with care, to punch a hole in a ¾-inch plate with a ½-inch punch, but the risk of the fracture of the punch necessitates in ordinary work that the punch should be rather larger in diameter than the thickness of the plates to be punched. Again, for each thickness of plates there is a diameter for the rivets, which is found as a general rule to make the soundest work. Practice is not quite uniform in this respect, but the following formulæ represent fairly the general usage in this respect.

For rivets passing through two thicknesses of plate, or a plate and cover, the diameter of the rivets would usually lie between

$$d = \tfrac{3}{4}t + \tfrac{5}{16}$$

and $d = \tfrac{7}{8}t + \tfrac{3}{8}$

the thickness of the plates ranging from $\frac{1}{4}$ to 1 inch. When several layers of plates are united:—

$$d = \tfrac{1}{8}t + \tfrac{5}{8} \quad \dots \quad \dots \quad (5)$$

Where $t$ is the total thickness to be rivetted, and ranges from 1 to 4 inches. Rivets of more than $1\frac{1}{4}$-inch diameter are very rarely employed, and in cases where a great thickness of plates is to be united, it is especially desirable that the plates should be drilled instead of punched, when, from the greater accuracy of the work, smaller rivets may be used than are desirable with punched plates. This was the course pursued with the Charing Cross Bridge, the bottom table of which is nearly 4 inches thick, and is rivetted with 1-inch rivets at 4-inch pitch.

If the diameter is thus arbitrarily fixed it will be found in general that if the bearing area is made sufficient to ensure durability, the shearing area will unavoidably be in excess of the absolute requirements of strength.

93. *Equal bearing and tearing resistance.* Putting $p$ for the pitch of the rivets across the plate or parallel to the joint (so that $p = b \div m$); $q$ for the number of rows of rivets on each side of the joint (so that $q = n \div m$); we may obtain the following convenient expression for the pitch of the rivetting of a joint in which the resistance of the bearing surface and the tearing area should be equal—

Resistance of tearing area corresponding to one longitudinal line of rivets $= (p - d)\, t\, s_1$

Resistance of bearing surface of rivets in that line $= q\, d\, t\, s_4$

Equating these expressions—

$$p = \left(\frac{q\, s_4}{s_1} + 1\right) d$$

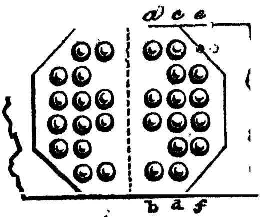
Let $s_1 = 5$; $s_4 = 7.5$.

$$p = (1.5\, q + 1)\, d$$

94. It is sometimes possible to save length and weight

in the cover plate by the arrangement shown in Fig. 67. In this case there are three rows of rivets on each side the joint. The plate, if it gave way by tearing, would yield at the section *e f*; the cover plate at the section *a b*. For though more metal is punched out in the section *c d*, the plate could not give way there without shearing at the same time the rivets in the section *e f*, or the cover plate without shearing the rivets in the section *a b*.

FIG. 67.




## JOINTS IN COMPRESSION.

95. The joints first used in the compression members of wrought-iron bridges were single rivetted butt joints, which have neither sufficient bearing area nor shearing area to transmit the whole stress which the plates are capable of carrying. These joints therefore depend for their security on the plates being so closely butted that the thrust is partially or wholly transmitted through the plane of contact of the plates to be joined. Could that contact always be secured, such joints would no doubt be adequate to retain the plates in position, and even in the case where the plates are only brought into contact by the yielding of the rivets after the completion of the bridge, the evil of deficient bearing surface is not so manifest in compression joints as in tension joints, because the direction of the thrust tends to keep the joint closed.

It is, however, very desirable that in the compression joints, as in the tension joints, the rivets and cover plate should be of sufficient strength to transmit the whole thrust, and that the contact of the butting plates should not be solely depended on. In many of the best-constructed bridges the compression joints are identical with the tension joints, and in all cases they should have the same amount of shearing and bearing area. The only way in which compression joints may safely differ from tension joints is that the rivets may be more

closely spaced across the plate, the quantity punched out in any section not affecting the strength of a compression joint as it does that of a tension joint. The minimum pitch of the rivetting is determined by the fact that holes cannot be punched very near to each other or to the edge of the plate, without the risk of cracking, or at all events of deteriorating the plate. The minimum distance edge to edge in bridge work should not be less than the diameter of the rivets, or the minimum pitch, centre to centre, less than twice the diameter. The pitch is not generally less than 3 inches.

# LECTURE VI.

## *ON ROOFS.*

96. In designing the supporting framework of roofs, precisely the same mechanical problem is presented as when a railway or roadway is to be carried over a ravine or river. Hence it is that the successive combinations adopted for bridges re-appear, in essentially the same forms, as roof principals. The stone vaulted inner roofs of some of the older churches are structurally identical with masonry bridges. Timber roof trusses are simply awkward-shaped girders, or, like the great roofs at King's Cross and over the transept of the first International Exhibition, they are timber arches analogous to those frequently erected as bridges in the earlier history of railways. Nor is the case otherwise with iron. All iron roofs may be classed as girders or as arches, with certain transitional forms which embody the features of both classes. And to pursue the analogy further, even the suspension principle, which at first sight, from the nature of the supports required, would seem inapplicable to the purpose, is, according to a proposal of M. M. Lehaitre and De Montdesir, to be pressed into the service of the roof builder.

97. Leaving, however, this last proposal, the first point which the iron roof designer must decide in any given case is, whether the principle of the arch or the girder is best suited to the circumstances of the case. And here, in roofs as in bridges, it will generally be found that where æsthetical considerations are of the first importance, recourse must be had to the arch. This form gives a pure outline, and adapts

itself readily to architectural requirements, and affords a clear space not disfigured by ties and struts, which, however. capable of noble treatment in wood, have hitherto been in iron invariably ugly, and with which the only thing to be done is, if possible, to hide them from view. On the other hand, if economical considerations prevail, some form of truss will probably be adopted, which in principle is essentially a girder.

The arched roof differs from the arched bridge in this, that whilst in the latter case we can easily accumulate in the abutments any amount of resistance to horizontal thrust, in the case of the roof, springing from the summit of lofty walls or pillars, horizontal resistance is very difficult to obtain. Hence, whilst the arched bridge may be of any required degree of flatness, the arched roof is restricted to those forms in which the thrust at the springing is a minimum. Iron arched roofs are therefore generally semi-circular, more rarely semi-elliptical. In a few instances a Gothic or ogival form has been adopted from considerations of appearance rather than from any special adaptation of the material to that form.

The trussed roof differs from the girder bridge in this, that whilst we can adopt for the bridge any outline which the economical distribution of the material requires, in the roof, the upper outline always and the lower often, is defined by considerations with which strength and economy have nothing to do.

98. *Classification of roofs.* Roofs may be divided, according to the principle on which they are constructed, into girder roofs and arched roofs; but in the different forms a gradation is found, which renders exact classification difficult. For the purpose of this Lecture it will be convenient to divide them into (1°) *Truss roofs*, with straight rafters, which are to be treated simply as braced frames or girders. (2°) *Bowstring roofs*, with curved rafters of small rigidity, and with a tie rod and bracing, which are to be dealt with also as braced polygonal frames. (3°) *Arched roofs*, in which the rigidity of the curved rafter is sufficient to resist the distorting influence of the load without additional bracing. Such roofs may or may not have a tie rod, but their proportions are to be determined on the principles applicable to the iron or elastic arch.

99. *Limits of stress.* As a matter of practice, whilst some Engineers restrict themselves to the same limits of stress in roofs as in bridges, viz., 5 tons per square inch in tension and 4 tons in compression, others permit stresses as high as 7 tons per square inch in the former. In France, 5 to 6 tons are allowed in tension and 5 tons in compression, limits which pretty closely agree with the limits for bridges, because the bar iron used in the construction of roofs has about 1-5th greater tensile resistance than the plate iron used for bridges. Roofs are said to have been constructed in America in which the calculated stress reaches 11 tons to the inch. But only the knowledge that the load had been over estimated can justify so high a limit. The use of a high limit of stress for roofs has indeed sometimes been defended on the ground that roofs are subject to a dead or quiescent load only. Such a defence involves an entire misconception. Of the elements which make up the load on roofs, one, amounting in ordinary cases to at least as much as all the others put together, is due to the pressure of the wind. This pressure, varying from moment to moment, with every change of force and direction in the wind, is essentially a live load, and there is no theoretical reason, from the nature of the loading, why a higher limit of stress should be adopted for roofs than for bridges. It is certainly true that roofs are more rarely tested to their extreme power of endurance, both because the wind only at rare intervals reaches its maximum velocity, and because it is only when the roof is struck broadside on that its force is most effectively exerted. But the security of an engineering structure ought not to depend on the mere chance of its escaping a contingency of this kind. Nor is it more scientific to over estimate the load and to strike a rough balance by taking a high limiting stress. The proper course is, to estimate with the greatest care the possible maximum load, and then to proportion the material in the same manner and for the same limiting stress as for a bridge, with this exception, that where the tenacity of the iron is known to be about 26 tons, the limiting stress in tension may be taken at 6 tons in place of 5.

100. *Load on roofs.* The load on a roof is of two kinds:

the permanent and the accidental load. The permanent load consists of—(*a*) The weight of the roof covering, of slate, tile, or metal, as the case may be, and its attachments; and the glazing, louvres, or other external appendages. (*b*) The weight of the upper framework, consisting of the longitudinal members or purlins, and if they exist, the subsidiary rafters resting on the purlins. (*c*) The weight of the supporting framework or roof principals, the trusses or arches by which the roof is sustained. The accidental load is the wind pressure or snow pressure, to which at times the roof may be subjected.

101. *Roof covering*. For iron roofs four kinds of covering have been employed. (1) A covering of slates attached by copper nails to wrought-iron laths or small angle irons, which in the smaller roofs serve the purpose of purlins, the principal rafters being spaced 6 to 7 feet apart. (2) A covering of plain or corrugated wrought iron on similar laths. The corrugation of the iron gives it considerable transverse strength, so that it may be used to span wider intervals than other coverings. Corrugated iron of only 16 W.G. in thickness, with flutes of 10 inches pitch and $2\frac{1}{2}$ inches deep, has been used spanning purlins 17 feet apart. (3) A covering of cast-iron plates has been used by Sir C. Barry for the roofs of the Houses of Parliament. In some of these the trusses are 2 feet 6 inches to 3 feet apart, and are covered by galvanised cast-iron plates $\frac{3}{8}$-inch thick, attached direct to the principals by screws. In other cases the principals are at a greater distance, and the plates are attached to purlins. This makes a strong, durable, and fire-proof covering, though costly and heavy. (4) The roof is boarded, and the boards covered with slates, with plain sheet iron about 20 W.G. in thickness, or with galvanised iron, or with sheet zinc.

It has been found that galvanised iron will not stand the chemical action of the acids precipitated from the atmosphere in large manufacturing towns. At Liverpool, the corrugated iron covering of the Tythebarn Street Station became unsafe to walk over, and required renewal in five years, and many other instances of rapid destruction might be quoted. Zinc

is more durable, but it has its own special defects, amongst which is its great expansibility with changes of temperature.

The following table gives the weight of ordinary roof coverings, with as much accuracy as is needed for estimating the load on a roof:—

*Weight of Roof Coverings.*

| | Lbs. per square foot. |
|---|---|
| Lead | 6 to 8 |
| Zinc | 1 „ $1\frac{3}{4}$ |
| Corrugated iron | 3 „ 4 |
| Slating | 8 |
| Pantiles | 10 |
| Plain tiles | 20 |
| Slates and iron laths | 10 |
| Sheet iron, 16 W.G., and laths | 5 |
| Corrugated iron and laths | $5\frac{1}{2}$ |
| Cast-iron plates, $\frac{3}{8}$-inch thick | 15 |
| Boarding and sheet iron, 20 W.G. | $6\frac{1}{2}$ |

102. *Weight of roof framing.* The roof framing consists—(1°) Of the upper or common rafters, when these are used spanning the purlins. (2°) Of the purlins, or longitudinal framing, which rest on the roof principals, and the spacing of which is determined in trussed roofs by the positions of the joints in the latter. It is necessary to avoid bending strains, that the purlins in such roofs should apply the load to the principals, as near the joints of the bracing as possible. (3°) Of the roof principals, the arches or girders, which support the load of the roof. In the following table the weights of some existing iron roofs have been gathered and arranged, to enable the Engineer to form a rough estimate of the probable load due to the weight of the framing, preparatory to commencing any calculations on the scantlings required in a roof. The weights are calculated per square foot of the projection of the roof on a horizontal plane, or per square foot of area covered by the roof.

## Weight of Roof Framing.

| | Clear Span. Ft. | Distance apart of Principals. Ft. | Weight per square foot of covered area in lbs. Of Purlins, &c. | Of Principal. | Total Ironwork. | Total with covering. |
|---|---|---|---|---|---|---|
| *Trussed Roofs.* | | | | | | |
| Pent ... ... | 15 | ... | ... | ... | 3.5 | ... |
| Common Truss. | 37 | 5 | 1.1 | 3.5 | 4.6 | 6.9 |
| ” | 40 | 12 | 2.0 | 3.5 | 5.5 | ... |
| ” | 54 | 14 | 6.5 | 3.0 | 9.5 | ... |
| ” | 55 | 6½ | 4.6 | 7.0 | 11.6 | ... |
| ” | 72 | 20 | 4.2 | 2.8 | 7.0 | ... |
| ” | 84 | 9 | 2.6 | 5.9 | 8.5 | ... |
| ” | 50 | 10 | ... | ... | 3.0 | 5.2 |
| ” | 100 | 14 | ... | ... | 7.0 | 9.0 |
| ” | 130 | 26 | 0.8 | 5.6 | 6.4 | 8.0 |
| ” | 140 | 12 | ... | 4.5 | ... | ... |
| *Bowstring Roofs.* | | | | | | |
| Manchester ... | 50 | 11 | ... | ... | 9.6 | ... |
| Lime Street ... | 154 | 26 | ... | 4.9 | ... | ... |
| Birmingham ... | 211 | 24 | ... | 7.3 | 11.0 | ... |
| *Arched Roofs.* | | | | | | |
| Small Corr. Iron | 40 | ... | ... | ... | ... | 2.5 |
| ” | 60 | ... | ... | ... | ... | 3.5 |
| Strasburg Rail.. | 97 | 13 | ... | ... | 12.0 | ... |
| Paris Ex. Large | 153 | 26 | 9.5 | 5.5 | 15.0 | ... |
| Dublin ... ... | 41 | 16 | 3.4 | 7.3 | 10.7 | ... |
| Derby ... ... | 81½ | 24 | 10.8 | 6.0 | 16.8 | ... |
| Sydenham ... | 120 | ... | 7.9 | 3.9 | 11.8 | ... |
| ” ... | 72 | ... | 8.4 | 2.9 | 11.3 | ... |
| St. Pancras ... | 240 | 29⅓ | 7.4 | 17.1 | 24.5 | ... |
| Cremorne ... | 45 | 14½ | 6.2 | 5.3 | 11.5 | ... |

103. *Accidental load.* The accidental load consists of the weight of accumulated snow, and of the wind pressure. For the former, from 5 to 20 lbs. per square foot of area covered has been allowed in different cases. But in this climate from 5 to 6 lbs. affords an ample security. The most various

estimates of the allowance for wind pressure will be found in different treatises, ranging from 6 to 40 lbs. per square foot; and the essential characteristic of this force is invariably overlooked, namely, that it is normal to the roof, instead of being, like the other elements of the load, vertical. It is of no use entering into refined mathematical analyses of the stresses in roofing, if such essential points are entirely ignored. In the large arched and braced roofs now frequently constructed, the wind pressure, from its unequal distribution, is the most dangerous of all the forces which the roof is required to resist.

The great influence of Tredgold has rendered practice more uniform than theory. Tredgold estimated the total accidental load on roofs at 40 lbs. per square foot of surface covered, or say 35 lbs. for wind and 5 lbs. for snow pressure, and the largest roofs are generally constructed for a uniformly distributed vertical load of this amount. When, however, the great Lime Street roof at Liverpool of 153 feet span was erected, it was immediately seen that a uniformly distributed load was not the worst casualty to which it might be subjected. Mr. Locke, therefore, in spite of a protest from its designer, required that a rib should be tested with a load of 40 lbs. per square foot hung on one half the roof, the other being unloaded, thus imitating, as nearly as was possible with a vertical load, the effect of the wind pressure acting broadside on. A rib of the great Birmingham roof was similarly tested, first with 40 lbs. per square foot uniformly distributed over the whole roof, and next with a similar load on half the roof only. In these cases the testing load represented the snow pressure, the wind pressure, and the weight of the permanent roof covering.

104. We are in this country visited annually by gales, singularly constant in their maximum force, amounting to from 20 to 25 lbs. per square foot on a surface perpendicular to their direction. More rarely, cyclonic storms sweep over the country, during which still higher pressures are registered. A pressure of 30 lbs. was registered during the Royal Charter storm; one of $33\frac{1}{2}$ lbs. was observed at Greenwich in the

storm preceding the fall of a station roof at Manchester. Higher pressures still, up to as much as 55 lbs., have been at various times recorded, but the accuracy of these observations is more doubtful. We shall probably allow margin enough for the worst contingency, if the maximum pressure of the wind is assumed at 40 lbs. per square foot of a surface perpendicular to its direction. On the inclined surface of a roof the pressure will be much less than this, the law of the variation of the pressure with the inclination being known with tolerable accuracy from the experiments of Hutton.

Let $P$ be the intensity of the wind pressure, in lbs. per square foot, on a surface perpendicularly opposed to it; $i$ the inclination of any plane surface to the wind's direction. Then the intensity of the pressure normal to the surface will be:—

$$P_n = P \sin i^{\,1.84 \cos i - 1}.$$

The component of that pressure, parallel to the wind's direction, will have the intensity:—

$$P_h = P \sin i^{\,1\cdot84 \cos i}.$$

The component perpendicular to the wind's direction, the intensity:—

$$P_v = P \cot i \sin i^{\,1\cdot84 \cos i}.$$

That is, if the wind blows horizontally $P_h$ is the horizontal and $P_v$ the vertical component of the pressure on the roof. Putting $P = 40$ we get the following values of the normal pressure and its components, for various inclinations of the roof surface to the direction of the wind:—

| | Lbs. per square foot of surface. | | |
|---|---|---|---|
| Angle of roof. | $P_n$ | $P_v$ | $P_h$ |
| 5° | 5 | 4·9 | 0·4 |
| 10 | 9·7 | 9·6 | 1·7 |
| 20 | 18·1 | 17·0 | 6·2 |
| 30 | 26·4 | 22·8 | 13·2 |
| 40 | 33·3 | 25·5 | 21·4 |
| 50 | 38·1 | 24·5 | 29·2 |
| 60 | 40·0 | 20·0 | 34·0 |
| 70 | 41·0 | 14·0 | 38·5 |
| 80 | 40·4 | 7·0 | 39·8 |
| [illegible] | 40·0 | 0 | 40·0 |

105. Now whether a roof is exposed to a vertical wind pressure of 35 lbs. per square foot, as in Tredgold's assumption, or not, it is certain that it will be exposed to the pressure of winds blowing horizontally. If, therefore, according to the common practice, the roof is designed to resist a uniform vertical pressure of 40 lbs. per square foot, plus the weight of the framing, it is at least equally necessary to examine whether it will resist the partial normal pressures given in the above table, which, in many cases, will produce a much greater distorting effect, and an entirely different distribution of stress on the bracing. It is difficult to fix the limits of the probable variation of the direction of the wind relatively to the roof, but if we suppose that in eddying gusts it may strike the roof, in any direction between the horizontal and vertical, then the maximum stress on any given member will be found in one or other of the three following cases:—

1° Wind blowing horizontally, which is the most ordinary condition of loading.

2° Wind normal to one side of the roof surface.

3° Wind vertical, which may possibly happen as a momentary condition, but which is certainly the least probable of possible modes of loading. The ordinary assumption, that the roof is subject to uniform vertical loads only, supposes the wind vertical, and neglects the horizontal component of the pressure, which will exist even in that case.

In the two former cases the loading due to the wind is unsymmetrically and unequally distributed. In the third case the loading is uniform in straight-raftered roofs, and symmetrical in arched roofs.

## TRUSSED ROOFS.

106. One of the simplest forms of iron truss is shown in

FIG. 68.

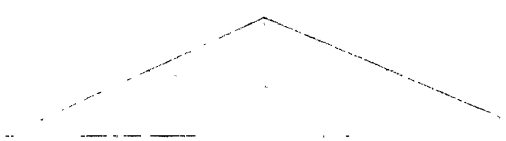


Fig. 68. It consists of two rafters, of a tie and king bolt, and of two struts supporting the rafters at mid length. Such a truss may be used for spans, up to about 30 feet.

Fig. 69 shows a better form of truss, introduced during the early period of railway construction, and which has since been almost universally adopted for roofs of moderate span.

FIG. 69.

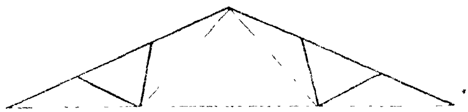


FIG. 70.

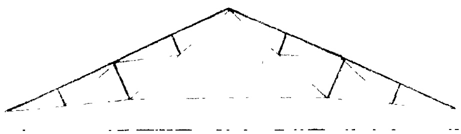


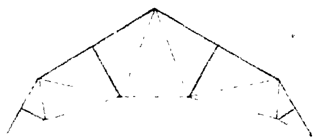
FIG. 71.

It consists of two rafters, each of which is supported by a strut, and the thrust of which is taken by a tie rod. With a uniform load the struts reduce the bending moment on the rafter, and with a partial load they distribute the stress. The merit of this truss is that the bracing is nearly all in tension. Mr. Bow has shown that if the members are proportioned to the stress it is more economical of material than any other form. When the span of the rafter becomes considerable, two struts may be used, as in Fig. 70, or secondary trussing may be introduced, as in Fig. 71. The rafters for moderate spans are T irons, and the ties and struts are bolted to the vertical rib. For larger spans, two angle irons, or better, two channel irons, may be used, placed back to back, with the ends of the bracing bars between them. In some cases the rafter is I-shaped. The rafters rest at the walls in cast-iron shoes, to which the tie rod is cottered. The ties are bars of round iron, and the struts commonly of cast iron, but preferably of T iron. The coupling of the ties and struts is effected by means

of small linking plates. The truss shown in Fig. 69 has been used for a span of 87 feet, and that shown in Fig. 71 for a span of 90 feet. It would, however, be better to restrict the use of these forms to spans not exceeding 60 feet. The trusses are usually 7 to 10 feet apart.

107. Fig. 72 shows the application of a precisely similar system of bracing to a mansard roof. This marks the first

FIG. 72.



transition towards the arched form, more fully developed in a roof over the Paris terminus of the Strasburg Railway. That roof consists of an arched rafter of considerable rigidity, $97\frac{1}{2}$ feet span and $29\frac{1}{2}$ feet rise, the thrust of which is taken by a tie rod, and which, to resist the distortion due to partial loading, is braced by light wrought-iron rods, in a manner very similar to the roofs already described.

108. Another form of truss, very commonly adopted for roofs of from 60 to 100 feet span, is derived from the wooden queen-post roof. Two modifications of this roof are shown in Figs. 73, 74. In the former the ties are vertical and the struts inclined; in the latter both struts and ties are inclined.

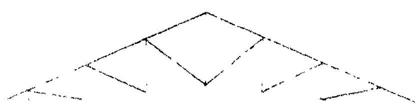


FIG. 73.

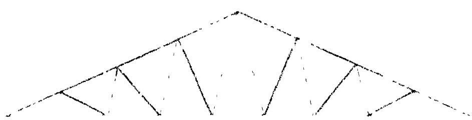


FIG. 74.

The latter type has the advantage that the struts are

shorter, more nearly perpendicular to the rafter, and less strained.

### BOWSTRING ROOFS.

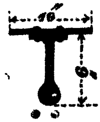
109. In 1847 the London and North-Western Railway Company required a roof for their station at Liverpool, to cover an area 153 feet 6 inches in width and 374 feet in length. In the case of railway stations there are special objections to intermediate supports. Not only do they occupy valuable space, and interfere with arrangements for shunting, but also from time to time, as traffic increases or is modified in character, re-arrangements of the lines and platforms become necessary, in ways impossible to foresee at the outset. The difficulty of effecting this, in stations with roofs of small span, may easily be imagined. It therefore occurred to Mr. Turner to cover the immense space at Liverpool with an iron roof in a single span. But a straight-raftered roof would have required an enormous height, in a span of that magnitude, and the walls afforded no adequate support for an arch. Mr. Turner, therefore, had recourse to a form similar to one of the most economical and convenient forms in bridge-work—the bowstring girder. The roof which he constructed (Fig. 75) consists of a series of segmental principals, spaced 21 feet 6 inches apart, with a wrought-iron tie rod and bracing. The arched rafter shown in section, in Fig. 76, consists of

FIG. 75.

a wrought-iron deck beam, 9 inches deep, with a plate 10 inches $\times \frac{1}{4}$ inch, rivetted to its upper flange. Towards the springing, this rib was strengthened by plates 7 inches $\times \frac{7}{8}$ inch, rivetted to the deck beam on each side. The struts are

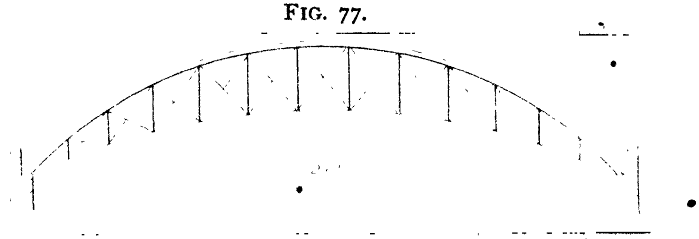
wrought-iron I-shaped bars, 7 inches deep. The tie rods are $6\frac{1}{2}$ square inches area, and the diagonal tension braces are $1\frac{1}{4}$ inch diameter. These trusses are fixed at one end, and at the other rest on rollers, permitting free expansion and contraction. The roof is covered partly with corrugated galvanised iron, No. 16 W.G., and partly with glass in large sheets, averaging 12 feet 4 inches × 3 feet 6 inches × $\frac{3}{8}$ inch. This admirable roof has only one defect, namely, that the counter-bracing applied to the centre bay has not been carried far enough, so that with extremely partial loads some of the ties may be thrown into compression. This defect has been remedied in later roofs by counter-bracing every compartment.

FIG. 76.



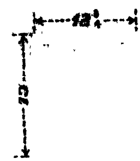
110. Over the New Street Station, Birmingham, a similar roof (Fig. 77), of still greater dimensions, was soon afterwards erected. This roof, $\frac{1}{6}$th of a mile in length, varies in span from 190 to 212 feet. It consists of bowstring principals, spaced 24 feet apart; the rise is $\frac{1}{5}$th the span, the tie rod having a versed sine of 17 feet, and the curved rafter a versed sine of $40\frac{1}{2}$ feet. The rafter is an I-shaped wrought-

FIG. 77.



iron beam (Fig. 78), 15 inches deep. The tie is a round bar in short lengths, 4 inches diameter, thickened at the junctions to which the bracing bars are attached, which are wrought-iron coupling boxes, into which the tie rods are screwed. The tension bars of the bracing are of plate iron, 5 inches to 3 inches in width and $\frac{5}{8}$ inch thick. A beautiful and effective form was adopted for the struts, which consist of four angle irons, arranged to form a cross, and spaced a small distance apart by cast-iron distance pieces. The angle irons are slightly bowed, so as to give the

FIG. 78.



strut a form swelling from the ends to the centre, which is both strong and pleasing. The purlins are of wood, 5 inches by 5 inches, trussed with a $\frac{7}{8}$ inch iron rod, and placed 8 feet apart. About half the roof is covered with corrugated iron, No. 18 W.G., and the other half with fluted rolled plate glass, $\frac{3}{16}$ inch thick, on wooden sash bars.

Of later roofs on the bowstring principle, the roof over the London Bridge terminus of the Brighton Railway may be mentioned. The span of this roof is 88 feet; rise, 27 feet; depth of truss from rafter to tie at centre, 18 feet. The principal trusses are 16 feet apart, with a light intermediate rib of trussed angle irons resting on the purlins. The rafter is I-shaped, as in the Birmingham roof; the tie is of $2\frac{1}{2}$-inch round iron, cottered to the rafter; the separate lengths of the tie rod are connected by wrought-iron linking plates, with pins $1\frac{3}{4}$ inches diameter, and to these plates the struts and diagonal ties are also attached. The struts are of wrought-iron tubing, 3 to 4 inches diameter. The purlins are I-shaped wrought-iron girders, about 10 feet apart. About half the roof surface is glazed, and the other half covered with zinc on $1\frac{1}{2}$-inch boarding.

In a bowstring roof over a railway station at Amsterdam, the curved rafter is of cast iron, of a section similar to the top boom of the Queensland Bridge (Plate III., Fig 1).

*Proportions of Bowstring Roofs.*

| | Clear Span. | Versed sine of Rafter. | Depth of Truss. | Distance apart of Principals. | Distance of Purlins. | Rafter. Depth. | Rafter. Width. | Rafter. Gross Area. | Section of Main Tie. |
|---|---|---|---|---|---|---|---|---|---|
| | Ft. | Ft. | Ft. | Ft. | Ft. | Ins. | Ins. | Sq. In. | Sq. In |
| Lime Street ...... | $153\frac{1}{2}$ | 30 | 12 | $21\frac{1}{2}$ | 12 | $9\frac{1}{4}$ | 10 | 14* | $6\frac{1}{2}$ |
| Birmingham ...... | 211 | $40\frac{1}{2}$ | 23 | 24 | 8 | 15 | $12\frac{3}{4}$ | 35 | $12\frac{1}{2}$ |
| Cannon Street ... | 190 | 60 | 30 | $33\frac{1}{2}$ | 11 | 21 | 14 | 28 | ... |
| Charing Cross ... | 164 | 45 | 20 | 35 | ... | 18 | $12\frac{1}{2}$ | 25 | ... |
| Blackfriars Bridge | $87\frac{1}{4}$ | 22 | 9 | $10\frac{3}{4}$ | ... | 6 | $10\frac{1}{4}$ | ... | ... |
| London Bridge ... | 88 | 27 | 18 | 16 | 12 | 7 | $6\frac{3}{8}$ | $10\frac{1}{2}$ | 5 |
| Amsterdam ........ | 120 | 30 | 13 | 25 | ... | ... | ... | ... | ... |

* At springing 25 square inches.

There is a method of covering wide spaces without intermediate supports, at less cost than by the adoption of roof principals of great span, which may be mentioned here. The method is to bridge the space to be covered by ordinary lattice girders, at moderate distances apart, and to carry the roofs upon these. This was the method adopted for the roofing of the great Exhibition of 1851, for that of the Covent Garden Opera House, and for that of the Pimlico Railway Station. At Covent Garden the lattice girders are 90 feet span, 9 feet deep, and 19½ feet apart, so that the roof trusses are only 19 feet span. At Pimlico the roofs are about 50 feet span.

## DETERMINATION OF STRESS IN TRUSSED ROOFS.

111. The complicated character of the loading of roofs renders the calculation of the stress by the methods applicable to braced girders very laborious. Each member of a roof is subject to a longitudinal tension or thrust, which is equal to the component of the load upon it, and the force applied by other bars at either extremity, resolved in its direction; and to flexure from the component of the load resolved at right angles to its direction (§ 7). The process of determining the stress in roofs consists therefore of two steps; (1) the determination of the longitudinal stresses; (2) the determination of the additional stress at certain points due to the flexure of the bars.

112. *Determination of longitudinal stress.* In ascertaining the longitudinal stress, the roof is, like the braced girder, considered as a perfectly jointed structure, and the loads are treated as if concentrated at the joints. In many roofs, the chief part of the load is actually applied close to the joints of the roof principal, by the purlins, and the load at each joint is to be found in precisely the same manner as the supporting forces of a beam (§ 5, 6). When the covering rests directly on the purlins, the load on each joint of the rafter of the principal may be taken to be equal to that of the part of the roof from centre to centre of the two adjoining bays. When the load rests on continuous rafters, resting in turn on the

purlins, it is more accurate to assume the proportions of the supporting forces as in § 6.

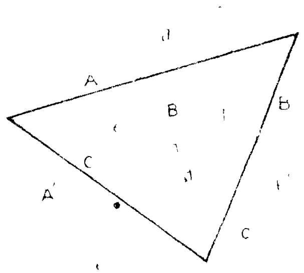
The methods given in text books for determining the longitudinal stresses, from the equivalent loads at the joints thus ascertained, proceed generally on the assumptions that the load is symmetrically distributed, and that it acts vertically, neither of which assumptions represents the actual conditions of the loading of the roof. The only useful method of algebraical calculation, when the load is unsymmetrical, is the method of the sections § 72, which although laborious is often useful, both in the original calculations and in checking the results of other methods. Fortunately, in the case of roofs, the special difficulty of a travelling load does not arise; to these structures, therefore, the method of a diagram of forces is peculiarly applicable. The diagrams generally given are, however, restricted by the erroneous assumptions above mentioned.

113. *Method of resolution of forces.* One graphic method, free from these objections, consists in the successive resolution of the forces at each joint by means of the parallelogram of forces. Beginning at the springing, the reaction at that point is resolved along the two bars terminating there. Proceeding to the nearest joint, the stress along one of the bars terminating at that joint, so ascertained, is combined with the load at that joint and the resultant resolved along two other bars terminating at the joint. The joints of the rafter and tie being taken in succession, the stresses along the last two bars, terminating in the other abutment, should, if the construction has been rightly carried out, balance the reaction at that point. This method is perfectly accurate in theory, but it is not so convenient in practice, because the errors due to slight inaccuracies of construction are cumulative.

114. *Professor Clerk Maxwell's diagram of forces.* If a principle could be found which, when once thoroughly understood, could be applied to determine the stresses in a roof, independently of any special assumptions as to the distribution of the loading, which in addition should give the stress on each member by the length of a single line, and which

should comprise a check on its own accuracy, so that if the diagram were wrongly constructed it would exhibit its own error, it cannot be doubted it would be of great service. Such a method is Professor Rankine's polygon of forces,* which has been stated in a somewhat more general form by Professor Clerk Maxwell.† Let *A B C* be a triangular jointed frame, in equilibrium, under the action of forces applied at the joints. We know from statics that in order that three forces may be in equilibrium their directions must meet in a point. Let *d e f* represent those directions. It is required to find the stresses in *A B C* due to the forces acting in the directions *d e f.* These forces being in equilibrium must be proportional to the sides of a triangle, respectively parallel to *d e f.* Draw *d' e' f'* parallel to *d e f.* Then *d' e' f'* represent the magnitudes of the forces acting at the joints of the frame. From the angles of the triangle so formed, draw *A' B' C'* parallel to *A B C.* It can be shown geometrically that these lines will

FIG. 79.




meet in one point. Then *A' B' C'* will represent the stresses on *A B C*, to the same scale as that on which *d' e' f'* represent

* "Applied Mechanics," § 148.

† "Philosophical Magazine," April, 1864, and a Paper read before the British Association for the Advancement of Science, in 1867. So far as plane frames are concerned, Professor Maxwell's differs from Professor Rankine's method only in the introduction of the conception of reciprocal figures, which is of great service in complicated frames. Figs. 79, 80, 81, are from Professor Maxwell's paper.

the forces acting at the joints. For since the force along $f$ is in equilibrium with the stresses along $B\,C$, and in the dotted figure that force is represented in magnitude and direction by $f'$, therefore the lines $B'\,C'$ parallel to the directions of $B\,C$ must represent, to the same scale, the forces in those directions which balance $f'$, or in other words the stresses on $B\,C$; and similarly with the forces at the other joints.

In the figure drawn in full, the lines $A\,B\,C$ represent the magnitudes and directions of the bars of the frame, and $d\,e\,f$ the directions of the forces acting at the joints. This will be called the *frame diagram*. In the dotted figure the lines represent the magnitudes, and are parallel to the directions of the forces acting on and in the frame. This will be called the *stress diagram*. Comparing the stress diagram with the frame diagram, it is easily seen that the lines in the latter which form a closed polygon are represented by lines in the former which meet at a point, and lines in the latter which meet at a point are represented by lines in the former which form a closed polygon. They are, therefore, *reciprocal figures*. Professor Maxwell has shown that in all cases where the load can be conceived to be applied to the joints of the external boundary of the frame, the condition that the forces are in equilibrium is the same as the condition that a reciprocal figure can be drawn. The method, therefore, contains a check on its own applicability. If in drawing the reciprocal figure the lines $A'\,B'\,C'$, representing the triangle $A\,B\,C$, had not closed up to a point, the figure must have been inaccurately drawn, or the magnitudes assumed for the external forces must have been such that equilibrium was not possible.

115. In certain cases it is not necessary in practice to draw all the lines of the reciprocal figure, and such cases present a slight difficulty. Suppose $A\,B\,C\,D$ (Plate VII., Fig. 1.) to be a jointed frame maintained in equilibrium by external forces acting at 1, 2, 3, 4. It is no longer necessary to equilibrium that these forces should meet in one point, and they may be supposed to meet in 5, 6. Then the polygon of forces 1-6, 4-5, 3-5, 2-6 in the stress diagram (Fig. 2) will

represent the magnitudes of the forces acting at 1, 2, 3, 4; but it no longer corresponds to a set of lines meeting in a point in the frame diagram. Suppose elastic strings introduced at 1, 2, 3, 4, maintained in tension by a string 5-6; the conditions of equilibrium and the forces acting on the frame will then remain unaltered. The line 5-6 in the stress diagram will be found to be parallel to 5-6 in the frame drawing, and the reciprocal figure is then complete, and conforms to the definition. In practice, when the external forces are known to be in equilibrium, the polygon of forces in the stress diagram may be drawn, and the lines *A B C D*, parallel to and representing the stress in the bars of the frame, without considering what lines are necessary to complete the reciprocal figure.

The diagram of stress having been drawn, the character of any given stress, whether tension or compression, is easily ascertained. Thus, suppose in Fig. 1, Plate VIII., the external forces to act towards the centre of the frame, as indicated by the arrows. In the stress diagram (Fig. 2), on any one of the lines corresponding to the external forces, as on 3-5, mark an arrow indicating its direction. Then, since the force on 3-5 is in equilibrium with the forces acting at 3 along $B$ and $C$, the arrows indicating the directions of the forces 3-5, $B_1$, $C_1$, must point the same way round the triangle; it is then easy to see that the forces along $B$, $C$, in equilibrium with 3-5 at 3, act towards the joint, and the stresses on $B$, $C$, must be thrusts.

116. *Application of method to a roof.* The following example, from Professor Maxwell's paper, will exhibit in a general form the application of the method of reciprocal figures to the determination of the stresses in roofs. Fig. 80 shows an ordinary form of roof truss. Suppose the load reduced to equivalent vertical loads $F_1$, $F_2$,......on the joints of the rafters; $G_1$, $G_2$,......on the joints of the tie. Let $P_1$, $P_2$, be the reactions of the side walls which, merely in order to avoid the overlapping of the lines, are shown out of the perpendicular. To draw the diagram of stress, Fig. 81, the polygon of external forces is first drawn by taking lines $F_1$, $F_2$,

......$P_2$, $P_1$, ......$G_1$, $G_2$, ......parallel to the corresponding lines in Fig. 81, and equal on any scale to the external forces taken in order round the frame. At the joint over the right-hand wall, the forces acting are $F_1$, $P_1$, $G_1$, and the stresses on $a_1$, $b_1$. These forces meeting in a point will be represented in the stress diagram by a polygon. Complete the polygon representing these in Fig. 81, by drawing lines parallel to $a_1$, $b_1$,

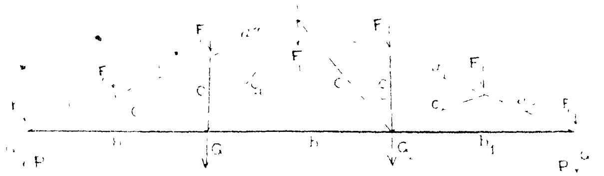


Fig. 80.

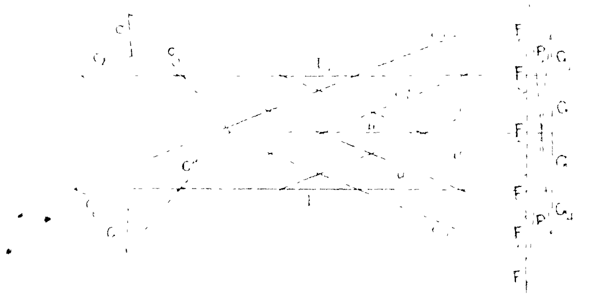


Fig. 81.

at the two unclosed extremities of the lines representing the other forces. At the nearest joint the forces acting are $F_2$ and the stresses on $a_1$, $a_2$, $c_1$; of these $F_2$ and the stress on $a_1$ are already drawn. Complete the polygon by drawing lines parallel to $a_2$, $c_1$. This process continued will give the stress diagram Fig. 82, and the accuracy of the process will be checked by the last line closing up between the required points to complete the last polygon. The lengths of the lines in Fig. 82 will then represent the stresses on the corresponding bars in Fig. 81.

## Examples of Diagrams of Stress.

117. In the following examples the diagram of stress has been drawn for two cases:—

*Case* 1. A uniformly distributed vertical load of 16 lbs. per square foot of the horizontal projection of the roof.

*Case* 2. A pressure normal to the slope, on one-half of the roof, of 20 lbs. per square foot of surface.

(*a*) The sum of the stresses thus obtained (or the difference if one is a thrust and the other a tension) will be the stress due to the permanent load and snow pressure (say 11 lbs. weight of framing and covering, and 5 lbs. snow), and to a wind acting horizontally, of such intensity that the pressure on a vertical surface would be about 40 lbs. per square foot. (*b*) The sum of the stress given in 1° and twice that in 2° is, nearly, the stress due to permanent load, snow pressure, and the same wind acting in a direction normal to the roof. (*c*) Lastly, three times the stress given in 1° will be the stress due to the permanent load, and a uniformly distributed vertical accidental load of 37 lbs. per square foot, such as has hitherto been generally assumed in designing roofs.

The roofs will be assumed to be 100 feet span, and the principals 10 feet apart. The slope will be 30°.

118. *Example I.—Case* 1. As the simplest possible case, a roof consisting only of two rafters and a tie may be taken, Plate VIII., Fig. 3. Under a uniform vertical load of 16 lbs. per square foot there will be, for 100 feet span and 10 feet width, 16,000 lbs., or 7·2 tons on each principal. Of this 1-4th, or 1·8 tons, will be supported at each abutment 1, 3, and 1-2nd, or 3·6 tons, at 2. The reaction at each abutment will be half the load, or 3·6 tons. On a vertical line of loads, therefore, set off downwards 1·8, 3·6, and 1·8; and upwards 3·6, 3·6. This line so divided represents the polygon of external forces. At 1 there are acting the reaction 3·6, the load 1·8, and the stresses on 1-3, 1-2. Two of these are already marked off on the line of loads. From their extremities complete the polygon of the forces at 1, in Fig. 4, by drawing the lines 1-3, 1-2, parallel to those bars in Fig. 3. The stress diagram is completed by drawing 2-3 parallel to 2-3.

119. *Case* 2. Suppose the wind blowing from the left on the rafter 1-2, whose length is 58 feet. The total pressure on that rafter will be 20 × 58 × 10 = 11,600 lbs. = 5·1 tons,

the

and the resultant will act in the direction of the dotted line 4-5, which bisects the rafter, and on measurement will be found to divide the tie in the ratio 1 to 2. Resolving this force first into two parallel forces at joints 1 and 2, 2·55 tons will act at each joint. Resolving again at 1 and 3, to find the reactions at those points, the reaction at 1 will be $\frac{2}{3} \times 5·1 = 3·4$ tons, and at 3, $\frac{1}{3} \times 5·1 = 1·7$ tons. Take in fig. 5 a line parallel to the normal wind pressure, and measure downwards the forces 2·55, 2·55, acting at 1 and 2, and then upwards, the reactions 3·4, 1·7. The line so divided is the load line. At 1 there are acting the load 2·55, the reaction 3·4, and the stresses on 1-2, 1-3. Complete the polygon of those forces, in fig. 5, by drawing the lines 1-2, 1-3, parallel to those bars in the frame. Lastly, 2-3 parallel to 2-3.

On fig. 3 have been marked against each bar the stresses thus obtained, in tons, the upper figure corresponding to fig. 4, and the lower to fig. 5. Since if the wind was on the right the stresses would be reversed, it is necessary to pay attention only to the maximum of the stresses on two corresponding members. Hence with the wind horizontal and the vertical load (§ 117. *a*), the stress on the rafters would be $3·6 + 2·9 = 6·5$ tons; and on the tie $3·1 + 1·7 = 4·8$ tons. With the wind normal and the vertical load (*b*) on the rafters $3·6 + 2 \times 2·9 = 9·4$ tons; and on the tie $3·1 + 2 \times 1·7 = 6·5$ tons. With a uniform vertical load of 48 lbs. per square foot (*c*), the stress on the rafters would be $3 \times 3·6 = 10·8$ tons; and on the tie $3 \times 3·1 = 9·3$ tons.

120. *Example II.—Case* 1. In Plate IX, fig. 1, another ordinary form of roof has been drawn. The total vertical load for 100 feet span and 10 feet width will be, as before, 7·2 tons nearly. If this load is assumed to be supported by each rafter as by a continuous beam (§ 6), the load on each bay of the rafters is $7·2 \div 4 = 1·8$ $(= w\,l)$; ~~the load supported at joints 1 and 7 will~~ be $\frac{3}{8} \times 1·8 = ·67$ tons; at 2 and 6, $\frac{5}{4} \times 1·8 = 2·25$ tons; and at 4, $2 \times \frac{3}{8} \times 1·8 = 1·34$ tons. The reaction at 1 and 7 will be $\frac{1}{2} \times 7·2 = 3·6$ tons. Setting off these quantities in order on the line of loads, fig. 2, the stress diagram may be drawn.

*Case* 2. With a normal pressure on one rafter of 20 lbs. per square foot the total load will be 5·3 tons, or 2·65 tons on each bay, giving for the loads at the joints of the rafter, considered as continuous, at 1, 1·0 ton; at 2, 3·3 tons; at 4, 1·0 ton. Resolving these parallel forces at 1 and 7, the reaction at 1 is 3·6 tons, and at 7 is 1·7 ton. In fig. 3 these quanities have been set off on the line of loads, and the diagram of stress has been drawn.

In fig. 1 the stresses thus obtained have been written down against each bar, the upper figure corresponding to the vertical and the lower to the normal load. Calculating out as in § 117, we get the following values of the stresses:—

| Bars. | Stress on bars in tons. Horizontal wind. (*a*) | Normal wind. (*b*) | Vertical load. (*c*) |
|---|---|---|---|
| *Rafters.* | | | |
| 1-2, 6-7 | 13·6 | 19·7 | 22·5 |
| 2-4, 4-6 | 12·5 | 18·6 | 19·2 |
| *Ties.* | | | |
| 1-3, 5-7 | 13·2 | 19·8 | 19·8 |
| 3-5 | 5·7 | 7·8 | 10·8 |
| *Bracing.* | | | |
| 2-3, 5-6 | 5·3 | 8·6 | 6·0 |
| 3-4, 4-5 | 9·8 | 14·4 | 9·6 |

121. *Example III.—Case* 1. In Plate X. the diagrams of stress have been drawn for a queen-post roof. With a uniform vertical load over the whole roof of 16 lbs. per square foot, or 7·2 tons altogether, the loads at the joints will be as follows:—at 1 and 15, ·35 ton; at 2 and 14, 1·03 ton; at 4 and 12, ·83 ton; at 6 and 10, 1·03 ton; at 8, ·70 ton. The reactions at 1 and 15 will be 3·6 tons. In fig. 2 these quantities have been set off on the vertical line of loads, and the diagram of stress has been drawn. The external forces meeting at 1 are the reaction 3·6, the load ·35, and the stresses on 1-2 and 1-3. These form the first polygon in the diagram of stress. The second is formed by drawing the lines parallel to 2-4 and 2-3; the third by drawing the lines parallel to 3-4, 3-5, and so on.

*Case* 2. With 20 lbs. wind pressure on one rafter, or 5·3

tons altogether, the loads at each joint, treating the rafter as continuous, will be:—at 1, ·51 ton; at 2 and 6, 1·5 ton; at 4, 1·21 ton; at 8, ·51 ton; the other joints being unloaded. Resolving these parallel normal forces at 1 and 15, the reaction at 1 is 3·5 tons, and that at 15 is 1·7 tons. Setting off these quantities on a line of loads perpendicular to the slope of the roof (fig. 3), the diagram of stress may be drawn as before. The numbering of the bars in fig. 1 being written against the lines representing the stress on those bars in fig. 3, the method of constructing the diagram is sufficiently indicated. It may be pointed out, however, that when joint 8 is reached, it will be found that the line 8-15 parallel to the unloaded rafter closes the figure, showing that with a normal pressure on one side of the roof the bracing bars on the other side are not strained.

The stresses thus found have been written against the bars in fig. 1, the upper figures corresponding to the vertical and the lower figures to the normal load. Calculating out as in § 117, we get the following values of the stresses:—

| Bars. | Stress on bars in tons. Horizontal Wind. (*a*) | Normal Wind. (*b*) | Vertical Load. (*c*) |
|---|---|---|---|
| *Rafters.* | | | |
| 1-2, 14-15 | 11·7 | 16·9 | 19·5 |
| 2-4, 12-14 | 11·8 | 17·4 | 18·6 |
| 4-6, 10-12 | 9·6 | 14·1 | 15·3 |
| 6-8, 8-10 | 7·7 | 11·3 | 12·3 |
| *Ties.* | | | |
| 1-3, 13-15 | 11·5 | 17·4 | 16·8 |
| 3-5, 11-13 | 9·2 | 13·7 | 14·1 |
| 5-7, 9-11 | 7·1 | 10·2 | 12·0 |
| 7-9 | 4·8 | 6·5 | 9·3 |
| *Bracing.* | | | |
| 2-3, 13-14 | 2·5 | 4·1 | 2·7 |
| 3-4, 13-12 | 3·1 | 5·1 | 3·3 |
| 4-5, 11-12 | 3·3 | 5·4 | 3·6 |
| 5-6, 10-11 | 3·5 | 5·7 | 3·9 |
| 6-7, 9-10 | 4·7 | 7·7 | 5·1 |
| 7-8, 8-9 | 4·8 | 7·8 | 5·4 |

It will be observed that, in this example, the stress on the bracing is greater when the direction of the wind is taken into account than when it is neglected.

122. *Relative economy of trusses.* An approximate estimate of the relative weights of the trusses shown in Plates VIII., IX., and X., may be obtained from the stresses given above. Let $s$ be the safe limit of stress in tons per square inch, $S$ the total stress on a bar in tons, $l$ its length in feet. Then it should theoretically have the section $S \div s$; its volume would then be $S l \div s$, in units of 12 cubic inches; and its weight would be $\frac{10}{3}\ S l \div s$ lbs. Taking $s = 3\frac{1}{2}$ for compressive stresses and $= 5$ for tensile stresses, and neglecting the stresses due to flexure, which have still to be investigated, we get for the above roof trusses, of 100 feet span and 10 feet apart:—

| | | Theoretical weight in lbs. of roof truss calculated for— | | |
|---|---|---|---|---|
| | | Horizontal wind. | Normal wind. | Vertical load. |
| Example I | ... | 1,080 | 1,472 | 1,813 |
| ,, II | ... | 3,752 | 3,816 | 3,918 |
| ,, III | ... | 2,345 | 3,578 | 3,446 |

As each bay of these roofs is 1,000 square feet area, the weight per square foot may be obtained by pointing off three figures in the above numbers. The actual will be greater than the theoretical weight, by an allowance for connections, and by the excess of weight of such bars as are not reduced to the dimensions theoretically necessary.

123. *Example* IV.—*Case* 1. *Bowstring roof.* Supposing still the roof of 100 feet span and the trusses 10 feet apart, the total load on the roof, at 16 lbs. per square foot, will be 7·2 tons. In a bowstring roof of 10 bays, this may be assumed to be distributed to the joints of the curved rafter, in the proportion of 0·36 ton over each springing and 0·72 ton at each of the other joints. The vertical reaction at each springing will be 3·6 tons. Setting off these quantities on the vertical line of loads, Plate XI., fig. 2, the diagram of stress may be drawn as before.

In bowstring roofs, as now constructed, the vertical bars of the bracing are struts, and the crossed diagonals are compa-

ratively thin plates, intended to act as ties. If we suppose that, from their form, the diagonal tie rods are incapable of resisting a compressive stress, then for each position of the loads, one of each pair of ties, which has the position of a strut, may be neglected, or supposed to be absent. Where all the three bracing bars meeting at a joint are ties, which happens sometimes when the main tie rod is arched, there is an ambiguity in determining the stress by the ordinary methods. There is no ambiguity in fact, for the stress on each bar will be proportional to its elongation by the deformation of the beam. The ambiguity is in theory only, and arises from the erroneous assumption that the material of the beam is *rigid* instead of being, as all materials in nature are, *elastic*. Nevertheless, to ascertain the stress on the three bars would involve formulæ of impracticable complexity; and it is necessary, in this case also, to suppose one of the three bars absent. Hence, before beginning to draw the diagram of stress for a bowstring roof, it is necessary to reduce the bracing to a single system of triangulation, by omitting half the diagonal ties. If, when this has been done, and the diagram of stress drawn, one of the diagonal bars is found in compression, that bar must be rejected, and the other of the two diagonals brought back. In Plate XI. the dotted lines show the diagonals which have been assumed absent in drawing the diagram of stress.

In fig. 2 only half the diagram of stress has been drawn, to avoid confusion. The load being symmetrical, the other half of the figure would be precisely similar to that which has been drawn. Tracing the directions of the forces at each joint, it will be found that, with a symmetrical load, the only bars in compression are the segments of the arched rafter and the third vertical from the springing on each side. The other verticals, as well as the diagonals and the segments of the main tie, are in tension.

124. *Case 2. Bowstring roof. Stresses due to wind.* Plate XII. In calculating the stresses due to the wind on a bowstring roof, the varying inclination of the surface struck should be taken into account. In the following example the

stresses have been calculated for a wind striking the roof horizontally. The load at each joint may be taken to be equal to the pressure due to the wind striking a surface whose area is equal to that portion of the roof supported by one bay of the rafter, inclined at the same angle as the tangent to the rib at the joint. The area corresponding to each bay of the rafter may be taken in this case at $10 \times 13 = 130$ square feet. The inclination of the rafter, measured from the drawing, is 65° at *A*; 45° at 1; 32° at 2; 20° at 3; 10° at 4. From the table, § 104, the pressures per square foot corresponding to those inclinations may be found to be 40, 36, 28, 18, and 10 lbs. respectively. At *A* the pressure is that due to half a bay, or $65 \times 40 = 2{,}600$ lbs. = 1·18 tons; at 1, $130 \times 36$ lbs. = 2·09 tons; at 2, similarly, 1·62 tons; at 3, 1·04 tons; at 4, 0·58 ton. Take radial lines through the joints, and measure on them $P_1 = 1·18$; $P_2 = 2·09$; $P_3 = 1·62$; $P_4 = 1·04$; $P_5 = ·58$. These lines represent the magnitudes and directions of the forces, the stresses due to which are required.

All bowstring roofs are supported so as to be on one side fast on the wall, at the other free to move on rollers. It is clear, therefore, that nearly the whole of the horizontal reaction resisting the horizontal component of the wind pressure must be applied at one abutment (§ 4), and this will be found to have a most important influence on the stress. Resolve $P_1$, $P_2$...... horizontally and vertically. The sum of the horizontal projections of those forces (represented by *a b* fig. 3) has been assumed in drawing the diagram of stress to act at *A*, or on the side towards which the wind is blowing, the end *B* being on rollers. In designing the roof a second diagram would require to be drawn for the contingency of the wind blowing on the other side of the roof which is free to move on rollers, and as the stresses would be found altogether different in the two cases, each bar of the roof would require to be strong enough to bear the stress arising in either case.

The next step is to find the vertical reactions. This may be conveniently effected by the method in § 5, shown as

applied to this example in Plate XII., fig. 2. Take $Oa$, $Ob$, $Oc$, $Od$, fig. 2, equal to the distances $A\,a$, $A\,b$, $A\,c$, $A\,d$, of the points, in which the directions of the forces $P_2$, $P_3$,......cut the chord line, from $A$; take $Oe$ equal to the span, and on a perpendicular at $e$, take $ef$, $eg$, $eh$, $ek$, equal to the vertical projections of the forces $P_1$, $P_2$......; through $f$, $g$, $h$, $k$, draw lines through $O$, cutting the corresponding vertical lines through $a$, $b$, $c$, $d$. Then the whole vertical reaction at $B$ is $al + bm + cn + dp$; and the difference between this and the sum of the vertical projections is the vertical component of the reaction at $A$.

The polygon of external forces, fig. 3, can now be drawn by taking $ac =$ the vertical reaction at $B$; $ab =$ the horizontal and $bd$ the vertical reaction at $A$; and the broken line from $c$ to $d$, consisting of segments equal and parallel to the lines representing the external forces $P_1$, $P_2$,...... The diagram of stress can then be constructed by drawing lines parallel to the bars at each joint in succession, as in previous instances. Fig. 3 is the complete stress diagram, and complicated as it may appear, is very easily constructed, if the principle of these diagrams is kept clearly in view.

125. The enormous difference in the magnitude and distribution of the stresses for a symmetrical and an unsymmetrical load, in roofs of this type, may be seen at a glance by comparing fig. 2, Plate XI., and fig. 3, Plate XII. The stresses on the bracing throughout are greater in the latter case, and the vertical bars, except the one nearest the springing at each end, are in compression. The diagonals are all in tension.

The importance of considering the direction of the wind pressure in these roofs may be further illustrated by considering the form the diagram would take when the direction of the wind was towards the side resting on the rollers. Without drawing another diagram with the forces acting on the other half of the roof, we may simply suppose the roof to be on rollers at $A$ and fast on the wall at $B$. Then the horizontal reaction represented by $a\,b$ (fig. 3) will be applied at the abutment $B$. The lines parallel to the segments of the

main tie, which in fig. 3 radiate from $a$, will in the new figure radiate from $b$. Thus $c\,e$ is parallel to the end of the rafter, and $b\,e$ to the segment of the main tie terminating at $B$, and represent the forces on those bars when the wind acts towards the free end of the roof. On tracing the forces round the polygon $a\,b\,e\,c$, it will be found that so far as the stresses due to the wind pressure are concerned, the last segment of the rafter is in this case in *tension*, and the last segment of the main tie in *compression;* a complete reversal of the ordinary condition of stress. By combining these stresses with those given in Plate XI., Fig. 2, due to the dead weight, we find that in the case of the end segment of the tie rod the compression due to the wind is not completely balanced by the tension due to the permanent load, but that that segment will actually be with the assumed loads slightly in compression.

126. If it be thought necessary to provide against a wind blowing otherwise than horizontally, the process of determining the stresses will be identically the same, except that in determining the intensity of the wind pressure on each bay the angle to be taken is that between the tangent to the rib at the joint and the assumed direction of the wind, instead of between the tangent and a horizontal line.

127. *Variation of stress at a section due to flexure.* When the forces applied to a bar are not actually applied close to the joints, and when at the same time they have a component transverse to the bar, the bar is bent as well as extended or compressed. The flexure increases the intensity of the stress on one side of any section, and diminishes it on the other, without varying the mean intensity on the whole section. But since the safety of the bar depends on the maximum and not on the mean intensity, the effect of flexure requires to be taken into account in estimating the scantlings required in roofing.

Flexure from the external load in the bars of the truss may be avoided by placing the purlins near the joints; but then the flexure of the bars due to their own weight will in roofing often require to be considered.

Let $M$ be the moment of flexure (§ 18) at any transverse section of a bar, in inch tons; $A$ the area in square inches, and $I$ the moment of inertia of the section; $T$ the longitudinal stress on the bar in tons, ascertained as above; $x$ the distance in inches from the neutral axis of the section to the edge of the beam convex to the axis of the bar, if $T$ is a thrust, and to the edge concave to the axis of the bar, if $T$ is a tension. Then the greatest intensity of stress on the section will be at the edge to which $x$ is measured, and will be equal to

$$s = \frac{T}{A} + \frac{M x}{I}$$

Where the first term on the right hand side of the equation represents the mean intensity of the stress on the section, and the second the increase of stress at the most strained edge, due to flexure. As the bars of a roof are generally of a uniform section, $s$ will be greatest at the section for which $M$ is greatest.

If the flexure is due to the weight of the bar, or to a uniform load, let $w$ be the weight or load in tons per inch of length; $\theta$ = the inclination of the bar to the horizon, $l$ = its length in inches. Then if the bar is freely supported at each end, the greatest bending moment is at the centre of the span, and

$$M = \tfrac{1}{8}\, w\, l^2 \cos.\, \theta$$

If, as more generally happens, the bar is part of a rafter or tie continuous over several joints, the greatest bending moment will generally be over a joint, and may be calculated on the same principle as the example in § 22.

## NOTE ON THE APPLICATION OF THE DIAGRAM OF FORCES TO THE DETERMINATION OF THE STRESSES IN BRIDGES.

The graphic method explained above may be applied to determine the stresses in braced girders not subject to a travelling load, with the same facility as to roofs. But in the case of girders subject to a rolling load, it is not so convenient as the methods already given, in the determination of the stresses on the bracing bars, because a separate diagram of stress would require to be drawn for every position of the rolling load. Sometimes, however, it is the easiest method of determining the stresses on the booms, which attain their maximum when the bridge is fully loaded. Plate XIII. shows this method applied to determine the stresses on the booms of the Warren girder, § 79, and the bowstring girder, § 81. In regard to fig. 2, the diagram of stress for the Warren girder, fig. 1, with 20 tons at each joint of the bottom boom and 10 tons over each abutment, it need only be observed that the stress on bar 7-5 is the length from *a* to *c;* that on 10-8, the length from *a* to *b;* that on 8-6, the length from *a* to *c*. The diagram is constructed, after marking off the load, by drawing the lines parallel to 11-9 and 11-10, completing with the lines representing the load and reaction at that joint, the polygon of forces at 11; next drawing 10-9, 10-8, completing the triangle of forces at 10; then the lines 9-8, 9-7, completing with the line representing the load at 9 the polygon of forces at that joint; and so on, whatever the number of bracing bars.

Plate XIII., Fig. 4, is the diagram of stress for the bowstring girder, Fig. 3. This girder is assumed to be 50 feet span, and the load is taken at 1·4 tons per foot run, as in § 81, giving 10 tons at each point of the bottom boom, and 5 tons carried directly at the abutments. Setting off the loads on a vertical line of loads, the stress diagram is constructed by drawing the lines parallel to 7-6, 7-8, so as to complete with the load of 3 tons and the reaction of 35 tons, the polygon of the forces meeting at 7. Then the lines parallel to 8-9, 8-6, are drawn, completing, with the line representing the stress on 7-8, the polygon of forces at 8. Next the lines parallel to

6-9, 5-6, with the lines representing the stress on 6-8, 6-7, complete the polygon of forces at 6; and so on.

The radiating lines represent the stress on the segments of the top boom; the horizontal lines those on the segments of the bottom tie; and the remaining lines the stress on the bracing bars in the case when the rolling load covers the whole bridge. Hence the stresses given by these diagrams for the bracing bars nearest the abutments (11-10, Fig. 2; 6-8, Fig. 4) should agree with and may be used to check the stresses found by other methods.

# APPENDIX.

## PRACTICAL EXAMPLES OF CALCULATION OF STRESS.

# EXAMPLE I.

## *PLATE GIRDER BRIDGE FOR SINGLE LINE OF RAILWAY.*

1. *Preliminary assumptions and data.* This bridge is assumed to have two main girders, with a platform of cross-girders between them.

Span, clear = 57 feet.

,, effective (assumed at nearly the distance centre to centre of bearings) = 60 feet.

Total length of girders = 64 feet.

Depth = 1-13th of span, say $4\frac{1}{2}$ feet, centre to centre of booms.

Width of booms = 1-40th of the span, say 18 inches.

Live load = $1\frac{1}{4}$ tons per foot run.

Limits of stress, 5 tons in tension, 4 tons in compression, $4\frac{1}{2}$ tons in shear, and 5 tons on the bearing surface, per square inch.

2. *Probable permanent load.* The total weight of platform girders may be taken as calculated in § 39 at 0·109 tons per foot run of bridge. The weight of ballast, rails, and timber sheeting, at 0·274 tons per foot run. Hence gross weight of platform is 0·383 tons per foot. The weight of the main girders may be found from the formulæ in § 40, taking from the table $C$ = 1,280, and for the mean stress on the gross section,

$$s = \tfrac{1}{2}(4 + \tfrac{5}{6} \times 5) = 4{\cdot}04$$

Weight per foot run of two main girders:—

$$= \frac{(2{\cdot}25 + 0{\cdot}383)\,60^2}{1280 \times 4{\cdot}04 \times 4{\cdot}5 - 60^2} = 0{\cdot}3 \text{ ton.}$$

Hence the load on each main girder is, in tons per foot run:—

$$\text{Dead load} = \tfrac{1}{2}(\cdot 383 + \cdot 3) = 0\cdot 341 = w_1$$
$$\text{Live load} = \tfrac{1}{2}(1\cdot 25) = \underline{0\cdot 625} = w_2$$
$$0\cdot 966$$

3. *Stresses on booms.* It will be sufficient to calculate the bending moments for 10 feet distances along the girder. They are given by the formula § 21 :—

$$M = (w_1 + w_2)\frac{x}{2}(l-x)$$

where $x$ is the distance of the section from either abutment. Dividing the bending moments by the depth of the girder, the total stress on each boom, in tons, is obtained.

| | Bending moment. | | Total stress on boom. | |
|---|---|---|---|---|
| At centre ... ... | 434·7 | feet tons. | 96·6 | tons. |
| 10 feet from centre | 386·4 | ,, | 85·6 | ,, |
| 20 feet from centre | 241·5 | ,, | 53·7 | ,, |
| At abutment ... | 0 | ,, | 0 | ,, |

4. *Shearing stress on web.* The shearing stress is due partly to a dead, partly to a travelling load. It is therefore given by the formula, § 14 :—

where $x$ is the distance from centre of girder.

| | Shearing force. | |
|---|---|---|
| At centre ... ... ... ... ... ... | 4·68 | tons. |
| 10 feet from centre ... ... ... ... | 11·73 | ,, |
| 20 feet from centre ... ... ... ... | 19·82 | ,, |
| At abutment ... ... ... ... ... | 28·95 | ,, |

5. *Centre section of booms.* The top boom will require a gross section at centre of 96·6 ÷ 4 = 24·15 square inches; and the bottom boom a net section of 96·6 ÷ 5 = 19·32 square inches. Designing the former first, the width being fixed at 18 inches, the section may be made up of:—

| | | Gross sectional area. | |
|---|---|---|---|
| Two angle irons, | $4\frac{1}{2}$ ins. × $4\frac{1}{2}$ ins. × $\frac{5}{8}$-in. | = 10·46 | sq. ins. |
| Two plates, | 18 ins. × $\frac{3}{8}$-inch | = 13·50 | ,, |
| | | 23·96 | ,, |

The total thickness to be rivetted in a boom of this form will be $1\frac{3}{8}$ inches, and for that thickness the rivets may be § 92, $\frac{1}{8} \times 1\frac{3}{8} + \frac{5}{8} = \frac{13}{16}$ nearly. For safely punching a $\frac{5}{8}$-inch angle iron, the rivets should not be less than $\frac{3}{4} \times \frac{5}{8} + \frac{5}{16} = \frac{13}{16}$ nearly. That diameter may therefore be taken. Using the same section as for the top boom, but deducting the area punched out for four holes in each plate and two in each angle iron, we get for the bottom boom:—

| | | Net sectional area. |
|---|---|---|
| Two angle irons, | $4\frac{1}{2} \times 4\frac{1}{2}$ ins. $\times \frac{5}{8}$-in. | = 8·43 sq. ins. |
| Two plates, | 18 ins. $\times \frac{3}{8}$-in | = 11·06 ,, |
| | | 19·49 ,, |

which is sufficient.

6. *Longitudinal section of booms.* It is obvious that the only reduction of section in the boom practically possible in this case is the omission of one of the two plates. The point at which that plate may be stopped off is most easily determined graphically, § 47. But if it is necessary to calculate it, we may proceed as follows:—The sectional area of the top boom, with only one plate, will be 10·46 + 6·75 = 17·21 square inches. Its resistance = 17·21 × 4 = 68·84 tons. Its moment of resistance 68·84 × 4½ = 309·78. Putting this value in the formula above for the bending moment:—

$$309{\cdot}78 = (w_1 + w_2) \frac{x}{2} (l-x)$$

giving

$$x^2 - 60\,x + 640 = 0$$
$$x = 30 \pm 16{\cdot}12$$
$$= 46{\cdot}12 \text{ or } 13{\cdot}88 \text{ feet}$$

distances which, measured from one end of the effective span, give the two points at which the section may be diminished by the omission of the second plate of the boom. A similar calculation should be made for the bottom boom.

7. *Design of web near abutment.* The total shear at the end of the girder is 28·95 tons, giving 28·95 ÷ 4½ = 6½ tons nearly per foot run of the vertical and horizontal sections at that point. There must be in the horizontal and vertical joints of the web 6½ ÷ 4½ = 1·44 square inches per foot run of shear-

ing section in the plates and rivets; and $6\frac{1}{2} \div 5 = 1.3$ inches of bearing area. Take provisionally the web plates at $\frac{3}{8}$-inch thick and the rivets $\frac{13}{16}$ diameter, as in the booms. The shearing area of a $\frac{13}{16}$ rivet is 0·52 square inches, and its bearing area in a $\frac{3}{8}$-inch plate is $\frac{13}{16} \times \frac{3}{8} = 0.30$ square inches. In a vertical joint of the web there will require to be $1.3 \div 0.3 = 4.3$ rivets per foot run that the bearing area may be sufficient; and as the rivets shear in two sections, there will require to be $1.44 \div 2\,(.52) = 1.4$ rivets per foot run that the shearing area may be sufficient. Taking five rivets to the foot there will be left in the plate $(12 \times \frac{3}{8}) - (\frac{13}{16} \times \frac{3}{8} \times 5) = 4.5 - 1.52 = 2.98$ square inches of shearing section per foot run, which is much more than sufficient. If, however, the plate is reduced in thickness, the bearing area of the rivets in the plate will be reduced, and the number of rivets to the foot, which is only a fraction more than sufficient to give the necessary bearing area with a $\frac{3}{8}$-inch plate, may require to be increased, which would make the pitch somewhat less than is desirable.

We proceed to examine the horizontal joint between the angle irons and boom. In this there will also be required 1·3 square inches of bearing area and 1·44 square inches of shearing area per foot run; and there are two rivets (one in each angle iron) in the width, each rivet shearing at one section. The bearing of the rivet in the angle iron is in this case $\frac{13}{16} \times \frac{5}{8} = .50$ square inches. There will be required, therefore, altogether $1.3 \div .5 = 3$ rivets per foot run to give the necessary bearing area, and $1.44 \div .52 = 3$ rivets to give the shearing area, or $1\frac{1}{2}$ rivets per foot run to each angle iron. It will be convenient, however, for other reasons to space the rivets rather closer, making the longitudinal pitch of the rivetting of the booms a multiple of that in the web, or say $2\frac{1}{2}$ rivets to the foot in each row.

8. *Joints of bottom boom.* Suppose a joint of the bottom boom to occur at 8 feet from the centre of the bridge. The stress at that point will be 89 tons. Of this the unbroken plate and angle irons will transmit a part proportional to their area. The net section of the broken plate being 5·53

square inches, the proportion of the stress to be carried by the joint will be $(5 \cdot 53 \div 19 \cdot 49)\ 89 = 25$ tons. There will be required, therefore, in the rivets on each side of the joint $25 \div 4\frac{1}{2} = 5 \cdot 5$ square inches of shearing area, and $25 \div 5 = 5$ square inches of bearing area. The shearing area of a $\frac{13}{16}$ rivet being $0 \cdot 52$ square inches, (if the joint is covered on one side only,) $5 \cdot 5 \div \cdot 52 = 11$ rivets will be necessary on each side of the joint, or, as there will be from the construction, four rivets in the width of the boom, three rows will be necessary. For bearing area, $5 \div \cdot 3 = 16$ rivets are necessary, which involves four rows of rivets on each side of the joint.

It would scarcely be possible to explain in detail the design of this girder, without working drawings, but the above calculations may be sufficient to indicate the principle on which the strength of every part of the girder is to be tested.

# EXAMPLE II.

## *LATTICE GIRDER BRIDGE FOR SINGLE LINE OF RAILWAY.*

1. The following calculations relate to a lattice girder bridge, Plate XIV., fig. 1, with two main girders, carrying cross-girders on their top. The stresses are calculated by Mr. J. H. Latham's method,* described in § 73.

2. *Preliminary assumptions and data.* Span, 230 feet clear. Total length of girders, 244 feet. Depth of girders between centres of booms = 20 feet. Width of booms, 3 feet 3 inches. Distance of main girders apart, 12 feet 9 inches, centre to centre. Length of cross-beams, on top of main girders, 17 feet.

The joints in the booms are 6 feet apart, and the angle at which the bracing is set is $\tan^{-1} \frac{20}{18} = 48°$ nearly.

The effective span is taken as equal to 39 bays or 234 feet.

| | | Tons. | cwt. | qrs. | lbs. |
|---|---|---|---|---|---|
| Total weight of ironwork (from drawings) | = | 347 | 0 | 0 | 0 |
| Planking ... ... ... ... ... | = | 22 | 6 | 0 | 0 |
| Ballast ... ... ... ... ... | = | 56 | 0 | 0 | 0 |
| Handrails ... ... ... ... | = | 2 | 14 | 0 | 0 |
| Rails and Chairs ... ... ... | = | 17 | 0 | 0 | 0 |
| Total dead weight of bridge ... | = | 445 | 0 | 0 | 0 |

This being the weight of the entire length of girders, the weight corresponding to length of effective span is

$$445 \times \tfrac{234}{244} = 427 \text{ tons.}$$

* The Construction of Wrought-iron Bridges. By J. H. Latham, M.A. 1858.

Or on each girder, 213·5 tons. This weight may be considered to be carried as follows:—

On top of each main girder—

Half weight of girder, weight of cross-beams, planking, ballast, handrails, rails and chairs = 136 tons.

On bottom of each main girder—

Half weight of girder = 77·5 tons.

Hence, dead load on each joint of top boom = 136 ÷ 39 = 3·487 tons.

Ditto ditto bottom „ = 77·5 ÷ 39 = 1·987 tons.

The live load is taken at about $1\frac{1}{2}$ tons per foot run, or exactly $173\frac{1}{4}$ tons on each girder for the length of the effective span. The loads for calculation are, therefore,

Load on each apex of bracing—

| | | |
|---|---|---|
| Dead load, top | = | 3·487 tons. |
| bottom | = | 1·987 „ |
| | | 5·474 „ |
| Live load, top | = | 4·442 „ |
| | | 9·916 „ |

Using Mr. Latham's notation,* we get, since the length of the bracing bars = $\sqrt{(18^2 + 20^2)}$ = 26·907 feet—

$$w_1 = 3{\cdot}487 \times \frac{26{\cdot}907}{20 \times 39} = 0{\cdot}1203$$

$$w_2 = 1{\cdot}987 \times \frac{26{\cdot}907}{20 \times 29} = 0{\cdot}0685$$

$$w_1' = 4{\cdot}442 \times \frac{26{\cdot}907}{20 \times 39} = 0{\cdot}1532$$

$$w_2' = 0$$

3. *Calculation of stresses on bars in compression.*

Stress on bar 19. (From inspection of sketch, Plate XIV., fig. 1.)

$$\begin{aligned} &{\cdot}1203\ (16 + 10 + 4 - 17 - 11 - 5.) \\ &+ {\cdot}0685\ (13 + 7 + 1 - 20 - 14 - 8 - 2.) \\ &+ {\cdot}1532\ (16 + 10 + 4.) \\ &= {\cdot}1203\ (-\ 3) + {\cdot}0685\ (-\ 23) + {\cdot}1532\ (30) = \\ &\quad 2{\cdot}6596 \text{ tons.} \end{aligned}$$

* It will be seen that $w$ in this calculation corresponds with $W$ cosec $\theta$ ÷ $l$ in § 73. The unit of length being the length of one bay.

Stress on bar 20 (by inspection of sketch).

$$\cdot 1203\,(17 + 11 + 5 - 16 - 10 - 4.)$$
$$+\ \cdot 0685\,(14 + 8 + 2 - 19 - 13 - 7 - 1.)$$
$$+\ \cdot 1532\,(17 + 11 + 5.)$$

Now, very little examination will show that the difference between the stress on this bar and that on the last bar is—

$$\begin{aligned} D_1 = \quad & \cdot 1203 \times 6 = .7218 \\ + & \cdot 0685 \times 7 = \cdot 4795 \\ + & \cdot 1532 \times 3 = \cdot 4596 \\ & \qquad\quad 1\cdot 6609 \text{ tons.} \end{aligned}$$

Hence the stress on this bar will be 2·6596 tons $+ D_1 =$ 2·6596 + 1·6609 = 4·3205 tons.

It is also easy, from examining the series of each of the terms representing the strain on the bar, to observe the law according to which the stresses on the bars change. We can, therefore, write down at once the stress on bar 21 by altering each of the arithmetical series in the same manner as we found it to change in passing from bar 19 to 20.

Stress on bar 21.

$$\cdot 1203\,(18 + 12 + 6 - 15 - 9 - 3.)$$
$$+\ \cdot 0685\,(15 + 9 + 3 - 18 - 12 - 6 - 0.)$$
$$+\ \cdot 1532\,(18 + 12 + 6.)$$

It is obvious that in this case $D_1$ has the same value as before or $D_2 = 0$.

Stress on bar 22.

$$\cdot 1203\,(19 + 13 + 7 + 1 - 14 - 8 - 2.)$$
$$+\ \cdot 0685\,(16 + 10 + 4 - 17 - 11 - 5.)$$
$$+\ \cdot 1532\,(19 + 13 + 7 + 1.)$$

Looking over these terms we shall see that $D_1$ is not the same as before, being in fact increased by the quantity $D_2 =$ ·1203 − ·0685 + ·1532 = ·2050 tons.

In writing down the stresses on the succeeding bars we shall write down only the significant terms of the series.

Stress on bar 23.

$$\cdot 1203\,(20\ldots\ldots 2 - 13\ldots\ldots 1.)$$
$$+\ \cdot 0685\,(17\ldots\ldots 5 - 16\ldots\ldots 4.)$$
$$+\ \cdot 1532\,(20\ldots\ldots 2.) \qquad\qquad D_2 = 0.$$

Stress on bar 24.

$$\cdot 1203\,(21 \ldots\ldots 3 - 12 \ldots\ldots 0.)$$
$$+\ \cdot 0685\,(18 \ldots\ldots 6 - 15 \ldots\ldots 3.)$$
$$+\ \cdot 1532\,(21 \ldots\ldots 3.) \qquad D_2 = 0.$$

Stress on bar 25.

$$\cdot 1203\,(22 \ldots\ldots 4 - 11 \ldots\ldots 5.)$$
$$+\ \cdot 0685\,(19 \ldots\ldots 1 - 14 \ldots\ldots 2.)$$
$$+\ \cdot 1532\,(22 \ldots\ldots 4.) \qquad D_2 = \cdot 0685 - \cdot 1203 = -\cdot 0518.$$

Stress on bar 26.

$$\cdot 1203\,(23 \ldots\ldots 5 - 10 \ldots\ldots 4.)$$
$$+\ \cdot 0685\,(20 \ldots\ldots 2 - 13 \ldots\ldots 1.)$$
$$+\ \cdot 1532\,(23 \ldots\ldots 5.) \qquad D_2 = 0.$$

Stress on bar 27.

$$\cdot 1203\,(24 \ldots\ldots 6 - 9 \ldots\ldots 3.)$$
$$+\ \cdot 0685\,(21 \ldots\ldots 3 - 12 \ldots\ldots 0).$$
$$+\ \cdot 1532\,(24 \ldots\ldots 6.) \qquad D_2 = 0.$$

Stress on bar 28.

$$\cdot 1203\,(25 \ldots\ldots 1 - 8 \ldots\ldots 2.)$$
$$+\ \cdot 0685\,(22 \ldots\ldots 4 - 11 \ldots\ldots 5.)$$
$$+\ \cdot 1532\,(25 \ldots\ldots 1.) \quad D_2 = \cdot 1203 - \cdot 0685 + \cdot 1532 = \cdot 2050.$$

Stress on bar 29.

$$\cdot 1203\,(26 \ldots\ldots 2 - 7 \ldots\ldots 1.)$$
$$+\ \cdot 0685\,(23 \ldots\ldots 5 - 10 \ldots\ldots 4.)$$
$$+\ \cdot 1532\,(26 \ldots\ldots 2.) \qquad D_2 = 0.$$

Stress on bar 30.

$$\cdot 1203\,(27 \ldots\ldots 3 - 6 \ldots\ldots 0.)$$
$$+\ \cdot 0685\,(24 \ldots\ldots 6 - 9 \ldots\ldots 3.)$$
$$+\ \cdot 1532\,(27 \ldots\ldots 3.) \qquad D_2 = 0.$$

Stress on bar 31.

$$\cdot 1203\,(28 \ldots\ldots 4 - 5).$$
$$+\ \cdot 0685\,(25 \ldots\ldots 1 - 8 \ldots\ldots 2.)$$
$$+\ \cdot 1532\,(28 \ldots\ldots 4.) \qquad D_2 = \cdot 0685 - \cdot 1203 = -\cdot 0518.$$

Stress on bar 32.

$$\cdot 1203\,(29 \ldots\ldots 5 - 4.)$$
$$+\ \cdot 0685\,(26 \ldots\ldots 2 - 7 \ldots\ldots 1.)$$
$$+\ \cdot 1532\,(29 \ldots\ldots 5.) \qquad D_2 = 0.$$

Stress on bar 33.

$$\cdot 1203\,(30 \ldots\ldots 6 - 3.)$$

+ ·0685 (27......3 − 6......0.)
+ ·1532 (30......6.) $D_2 = 0$.

Stress on bar 34.
·1203 (31......1 − 2.)
+ ·0685 (28......4 − 5.)
+ ·1532 (31......1.) $D_2 = ·1203 − ·0685 + ·1532 = ·2050$.

Stress on bar 35.
·1203 (32......2 − 1.)
+ ·0685 (29......5 − 4.)
+ ·1532 (32......2.) $D_2 = 0$.

Stress on bar 36.
·1203 (33......3 − 0.)
+ ·0685 (30......6 − 3.)
+ ·1532 (33......3.) $D_2 = 0$.

Stress on bar 37.
·1203 (34......4.)
+ ·0685 (31......1 − 2.)
+ ·1532 (34......4.) $D_2 = ·0685 − ·1203 = − ·0518$.

Stress on bar 38.
·1203 (35......5.)
+ ·0685 (32......2 − 1.)
+ ·1532 (35......5.) $D_2 = 0$.

Stress on bar 39.
·1203 (36......6.)
+ ·0685 (33......3 − 0.)
+ ·1532 (36......6.) $D_2 = 0$.

Stress on bar 40.
·1203 (37......1.)
+ ·0685 (34......4.)
+ ·1532 (37......1.) $D_2 = ·1203 − ·0685 + ·1532 = ·2050$.

Stress on bar 41.
·1203 (38......2.)
+ ·0685 (35......5.)
+ ·1532 (38......2.) $D_2 = 0$.

Starting from the value of the stress on bar 19 (2·6596 tons), already obtained, and adding successively the values of the first and second differences thus obtained, we get for the stresses on the bars the numbers in the following table:—

*Compressive stresses on bars.*

| No. of bar. | Compression in tons. | $D_1$ | $D_2$ |
|---|---|---|---|
| 19 | 2·6596 | | |
| 20 | 4·3205 | 1·6609 | 0 |
| 21 | 5·9814 | 1·6609 | ·2050 |
| 22 | 7·8473 | 1·8659 | 0 |
| 23 | 9·7132 | 1·8659 | 0 |
| 24 | 11·5791 | 1·8659 | − ·0518 |
| 25 | 13·3932 | 1·8141 | 0 |
| 26 | 15·2073 | 1·8141 | 0 |
| 27 | 17·0214 | 1·8141 | ·2050 |
| 28 | 19·0405 | 2·0191 | 0 |
| 29 | 21·0596 | 2·0191 | 0 |
| 30 | 23·0787 | 2·0191 | − 0·518 |
| 31 | 25·0460 | 1·9673 | 0 |
| 32 | 27·0133 | 1·9673 | 0 |
| 33 | 28·9806 | 1·9673 | ·2050 |
| 34 | 31·1529 | 2·1723 | 0 |
| 35 | 33·3252 | 2·1723 | 0 |
| 36 | 35·4975 | 2·1723 | − ·0518 |
| 37 | 37·6180 | 2·1205 | 0 |
| 38 | 39·7385 | 2·1205 | 0 |
| 39 | 41·8590 | 2·1205 | ·2050 |
| 40 | 44·1845 | 2·3255 | 0 |
| 41 | 46·5090 | 2·3255 | |
| Stress on 3 bars terminating at abutment | 132·5525 | | |

4. *Calculation of stresses on bars in tension.* In a precisely similar manner we obtain the stresses on the bars in tension.

Stress on bar 25 (by inspection).

$$\begin{aligned} &\cdot 1203\,(11 + 5 - 22 - 16 - 10 - 4.) \\ &+ \cdot 0685\,(14 + 8 + 2 - 19 - 13 - 7 - 1.) \\ &+ \cdot 1532\,(11 + 5.) \\ &\qquad = \cdot 1203 \times (-36) + \cdot 0685 \times (-16) + \cdot 1532 \\ &\qquad\quad \times 16 = -2\cdot 9756. \end{aligned}$$

The negative sign shows that this bar does not in any position of the travelling load come into tension.

Stress on bar 24.

$$\cdot 1203\,(12 + 6 - 21 - 15 - 9 - 3.)$$
$$+ \cdot 0685\,(15 + 9 + 3 - 18 - 12 - 6 - 0.)$$
$$+ \cdot 1532\,(12 + 6.)$$
$$D_1 = 6 \times \cdot 1203 + 7 \times \cdot 0685 + 2 \times \cdot 1532 = 1.5077.$$

Then stress on bar $= - 2\cdot 9756 + 1\cdot 5077 = - 1\cdot 4679$. Also a thrust.

Stress on bar 23.

$$\cdot 1203\,(13 + 7 + 1 - 20 - 14 - 8 - 2.)$$
$$+ \cdot 0685\,(16 + 10 + 4 - 17 - 11 - 5.)$$
$$+ \cdot 1532\,(13 + 7 + 1.)$$
$$D_2 = \cdot 1203 - \cdot 0685 + \cdot 1532 = \cdot 2050.$$
$$D_1 = 1\cdot 5077 + \cdot 2050 = 1\cdot 7127.$$

Stress on bar $= - 1\cdot 4679 + 1\cdot 7127 = \cdot 2448$ tons.

The sign having changed, this is the first bar which comes into tension.

Stress on bar 22.

$$\cdot 1203\,(14 + 8 + 2 - 19 - 13 - 7 - 1.)$$
$$+ \cdot 0685\,(17 + 11 + 5 - 16 - 10 - 4.)$$
$$+ \cdot 1532\,(14 + 8 + 2.) \qquad D_2 = 0.$$

Stress on bar 21.

$$\cdot 1203\,(15 \ldots\ldots 3 - 18 \ldots\ldots 0.)$$
$$+ \cdot 0685\,(18 \ldots\ldots 6 - 15 \ldots\ldots 3.)$$
$$+ \cdot 1532\,(15 \ldots\ldots 3.) \qquad D_2 = 0.$$

Stress on bar 20.

$$\cdot 1203\,(16 \ldots\ldots 4 - 17 \ldots\ldots 5.)$$
$$+ \cdot 0685\,(19 \ldots\ldots 1 - 14 \ldots\ldots 2.)$$
$$+ \cdot 1532\,(16 \ldots\ldots 4.) \qquad D_2 = \cdot 0685 - \cdot 1203 = - \cdot 0518.$$

Stress on bar 19.

$$\cdot 1203\,(17 \ldots\ldots 5 - 16 \ldots\ldots 4.)$$
$$+ \cdot 0685\,(20 \ldots\ldots 2 - 13 \ldots\ldots 1.)$$
$$+ \cdot 1532\,(17 \ldots\ldots 5.) \qquad D_2 = 0.$$

Stress on bar 18.

$$\cdot 1203\,(18 \ldots\ldots 6 - 15 \ldots\ldots 3.)$$
$$+ \cdot 0685\,(21 \ldots\ldots 3 - 12 \ldots\ldots 0.)$$
$$+ \cdot 1532\,(18 \ldots\ldots 6.) \qquad D_2 = 0.$$

Stress on bar 17.

$\cdot 1203\ (19\ldots\ldots 1 - 14\ldots\ldots 2.)$
$+ \cdot 0685\ (22\ldots\ldots 4 - 11\ldots\ldots 5.)$
$+ \cdot 1532\ (19\ldots\ldots 1.)$ $D_2 = \cdot 1203 - \cdot 0685 + \cdot 1532 = \cdot 2050.$

Stress on bar 16.

$\cdot 1203\ (20\ldots\ldots 2 - 13\ldots\ldots 1.)$
$+ \cdot 0685\ (23\ldots\ldots 5 - 10\ldots\ldots 4.)$
$+ \cdot 1532\ (20\ldots\ldots 2.)$ $D_2 = 0.$

Stress on bar 15.

$\cdot 1203\ (21\ldots\ldots 3 - 12\ldots\ldots 0.)$
$+ \cdot 0685\ (24\ldots\ldots 6 - 9\ldots\ldots 3.)$
$+ \cdot 1532\ (21\ldots\ldots 3.$ $D_2 = 0.$

Stress on bar 14.

$\cdot 1203\ (22\ldots\ldots 4 - 11\ldots\ldots 5.)$
$+ \cdot 0685\ (25\ldots\ldots 1 - 8\ldots\ldots 2.)$
$+ \cdot 1532\ (22\ldots\ldots 4.)$ $D_2 = \cdot 0685 - \cdot 1203 = - \cdot 0518.$

Stress on bar 13.

$\cdot 1203\ (23\ldots\ldots 5 - 10\ldots\ldots 4.)$
$+ \cdot 0685\ (26\ldots\ldots 2 - 7\ldots\ldots 1.)$
$+ \cdot 1532\ (23\ldots\ldots 5.)$ $D_2 = 0.$

Stress on bar 12.

$\cdot 1203\ (24\ldots\ldots 6 - 9\ldots\ldots 3.)$
$+ \cdot 0685\ (27\ldots\ldots 3 - 6\ldots\ldots 0.)$
$+ \cdot 1532\ (24\ldots\ldots 6.)$ $D_2 = 0.$

Stress on bar 11.

$\cdot 1203\ (25\ldots\ldots 1 - 8\ldots\ldots 2.)$
$+ \cdot 0685\ (28\ldots\ldots 4 - 5.)$
$+ \cdot 1532\ (25\ldots\ldots 1.)$ $D_2 = \cdot 1203 - \cdot 0685 + \cdot 1532 = \cdot 2050.$

Stress on bar 10.

$\cdot 1203\ (26\ldots\ldots 2 - 7\ldots\ldots 1.)$
$+ \cdot 0685\ (29\ldots\ldots 5 - 4.)$
$+ \cdot 1532\ (26\ldots\ldots 2.)$ $D_2 = 0.$

Stress on bar 9.

$\cdot 1203\ (27\ldots\ldots 3 - 6.)$
$+ \cdot 0685\ (30\ldots\ldots 6 - 3.)$
$+ \cdot 1532\ (27\ldots\ldots 3.)$ $D_2 = 0.$

Stress on bar 8.

$\cdot 1203\ (28\ldots\ldots 4 - 5.)$

+ ·0685 (31......1 − 2.)
+ ·1532 (28......4.) $D_2 =$ ·0685 − ·1203 = − ·0518.

Stress on bar 7.

·1203 (29......5 − 4.)
·0685 (32......2 − 1.)
·1532 (29......5.) $D_2 = 0$.

Stress on bar 6.

·1203 (30......6 − 3.)
·0685 (33......3 − 0.)
·1532 (30......6.) $D_2 = 0$.

Stress on bar 5.

·1203 (31......1 − 2.)
·0685 (34......4.)
·1532 (31......1.) $D_2 =$ ·1203 − ·0685 + ·1532 = ·2050.

Stress on bar 4.

·1203 (32......2 − 1.)
+ ·0685 (35......5.)
+ ·1532 (32......2.) $D_2 = 0$.

Stress on bar 3.

·1203 (33......3 − 0.)
+ ·0685 (36......6.)
+ ·1532 (33......3.) $D_2 = 0$.

Stress on bar 2.

·1203 (34......4.)
+ ·0685 (37......1.)
+ ·1532 (34......4.) $D_2 =$ ·0685 − ·1203 = − ·0518.

Stress on bar 1.

·1203 (35......5.)
+ ·0685 (38......2.)
+ ·1532 (35......5.) $D_2 = 0$.

So that, as before, we get the tensions on the bars given in the following table:—

*Tensile stresses on bars.*

| No. of bar. | Tension in tons. | $D_1$ | $D_2$ |
|---|---|---|---|
| 23 | 0·2448 | | |
| 22 | 1·9575 | 1·7127 | 0 |
| 21 | 3·6702 | 1·7127 | − ·0518 |

| No. of bar. | Tension in tons. | $D_1$ | $D_2$ |
|---|---|---|---|
| 20 | 5·3312 | 1·6609 | 0 |
| 19 | 6·9920 | 1·6609 | 0 |
| 18 | 8·6529 | 1·6609 | ·2050 |
| 17 | 10·5188 | 1·8659 | 0 |
| 16 | 12·3847 | 1·8659 | 0 |
| 15 | 14·2506 | 1·8659 | – ·0518 |
| 14 | 16·0647 | 1·8141 | 0 |
| 13 | 17·8788 | 1·8141 | 0 |
| 12 | 19·6929 | 1·8141 | ·2050 |
| 11 | 21·7120 | 2·0191 | 0 |
| 10 | 23·7311 | 2·0191 | 0 |
| 9 | 25·7502 | 2·0191 | – ·0518 |
| 8 | 27·7175 | 1·9673 | 0 |
| 7 | 29·6848 | 1·9673 | 0 |
| 6 | 31·6521 | 1·9673 | ·2050 |
| 5 | 33·8244 | 2·1723 | 0 |
| 4 | 35·9967 | 2·1723 | 0 |
| 3 | 38·1690 | 2·1723 | – ·0518 |
| 2 | 40·2895 | 2·1205 | 0 |
| 1 | 42·4100 | 2·1205 | |
| Stress on 3 bars terminating at abutment. | 120·8685 | | |

5. *Check on calculations.* We have a means of checking these results. The strain on the end bars must be equal to the shearing force at the abutment resolved in their direction. Now the shearing force at the abutment is half the distributed load, or $\frac{1}{2}$ (136 + 77·5 + 173·25) = 193·37. Out of this a load, equivalent to weight on one bay, is carried directly at the abutments, because an apex top and bottom falls directly over the abutment. The shearing stress on the bars terminating at the abutment is, therefore, 193·37 – 4·958 = 188·4 tons, and this resolved along bars

$$= 188 \cdot 4 \times \frac{26 \cdot 907}{20} = 253 \text{ tons}$$

the total stress on the bars. Now the actual total stress on the three last compression bars and the three last tension

bars from the tables is 120·87 + 132·55 = 253·42 tons, which is as near as could be expected.

6. *Dimensions actually adopted.* The actual dimensions on the struts and ties in this bridge are given in the following tables:—

*Compression bars.*

| | Two angle irons. | | |
|---|---|---|---|
| From centre to bar 24 | $4 \times 3 \times \frac{7}{16}$ | with $1-1''$ | rivet* |
| ,, bar 25 to bar 27 | $4\frac{1}{2} \times 3 \times \frac{1}{2}$ | ,, $1-1''$ | ,, |
| ,, ,, 28 ,, 30 | $5 \times 3 \times \frac{5}{8}$ | ,, $1-1''$ | ,, |
| ,, ,, 31 ,, 33 | $5\frac{1}{2} \times 3\frac{1}{2} \times \frac{11}{16}$ | ,, $1-1''$ | ,, |
| ,, ,, 34 ,, 37 | $6 \times 4 \times \frac{3}{4}$ | ,, $2-\frac{7}{8}''$ | ,, |
| ,, ,, 38 ,, 41 | $7 \times 4 \times \frac{3}{4}$ | ,, $2-\frac{7}{8}''$ | ,, |

*Tension bars.*

| | Two bars. | | |
|---|---|---|---|
| From centre to bar 18 | $4 \times \frac{5}{8}$ | with $1-\frac{7}{8}''$ | rivet |
| ,, bar 19 to bar 14 | $4 \times \frac{3}{4}$ | ,, $1-\frac{7}{8}''$ | ,, |
| ,, ,, 15 ,, 11 | $4\frac{1}{2} \times \frac{3}{4}$ | ,, $1-\frac{7}{8}''$ | ,, |
| ,, ,, 12 ,, 7 | $5 \times \frac{3}{4}$ | ,, $1-\frac{7}{8}''$ | ,, |
| ,, ,, 8 ,, 3 | $7 \times \frac{3}{4}$ | ,, $2-\frac{7}{8}''$ | ,, |
| ,, ,, 4 ,, 1 | $7\frac{1}{2} \times \frac{3}{4}$ | ,, $2-\frac{7}{8}''$ | ,, |

The rivets, as given above, are the rivets in one section across the bar, whose area, therefore, is to be deducted from the area of the bar. Calculating the intensity of the stress from the above data, for the bars on which the intensity is greatest, that is, the bars at those points at which they change dimensions, we get the following results:—

*Comparison of intensity of stresses on bars.—Compression bars.*

| No. of bar. | Gross area of Bars. Square inches. | Area of rivet holes. Square inches. | Net area of bars. Square inches. | Total compression in tons. | Compression per square inch in tons. |
|---|---|---|---|---|---|
| 24 | 5·6884 | ·875 | 4·8134 | 11·5791 | 2·460 |
| 27 | 7·0000 | 1·000 | 6·0000 | 17·0214 | 2·837 |
| 30 | 9·2200 | 1·250 | 7·9700 | 23·0787 | 2·896 |
| 33 | 11·4262 | 1·375 | 10·0512 | 28·9806 | 2·678 |
| 37 | 13·8800 | 2·624 | 11·2560 | 37·6180 | 3·155 |
| 41 | 15·3800 | 2·624 | 12·7560 | 46·5090 | 3·640 |

* That is, not the total number of rivets, but the number in any one section of bar.

It will be observed that it is impossible to reduce the bars towards the centre in the theoretical proportion, the result is that the intensity of the stress increases towards the centre. de

*Tension bars.*

| No. of tie. | Area of two bars, square inches. | Area of rivet holes. square inches. | Net area. square inches. | Total tension in tons. | Tension per square inch in tons. |
|---|---|---|---|---|---|
| 18 | 5·000 | 1·090 | 3·910 | 8·6529 | 2·194 |
| 14 | 6·000 | 1·312 | 4·688 | 16·0647 | 3·397 |
| 11 | 6·750 | 1·312 | 5·438 | 21·7120 | 3·965 |
| 7 | 7·500 | 1·312 | 6·188 | 29·6848 | 4·763 |
| 3 | 10·500 | 2·624 | 7·876 | 38·1690 | 4·819 |
| 1 | 11·250 | 2·624 | 8·626 | 42·4100 | 4·884 |

7. *Stresses on booms.* To calculate the stresses along the booms the most accurate method is to proceed in a manner analogous to that adopted in calculating the stresses on the bars. But for practical purposes it is generally more convenient to use the formula already explained.

$$2$$

In this case $w_1 + w_2 = \dfrac{388}{237} = 1·637$ and $c = \dfrac{237}{2} = 118·5$.

Hence bending moment at centre of girder ($x = 0$) is

$$M_0 = \frac{1·637 \times 118·5^2}{2} = 11493 \text{ foot tons.}$$

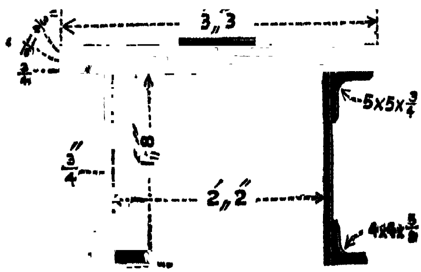
And total thrust on top boom, or tension on bottom boom, equal to bending moment divided by depth of girder $= \dfrac{11493}{20} = 574·65$ tons.

Similarly calculating the bending moment and stress on booms for 15 feet distances from centre of girder, we get:—

| | Bending moment. | Total stress on boom. |
|---|---|---|
| At centre ... ... | 11493 foot tons ... | 574·65 tons. |
| 15 feet from centre | 11309 ,, ... | 565·45 ,, |
| 30 feet ,, ,, | 10757 ,, ... | 537·85 ,, |
| 45 feet ,, ,, | 9836 ,, ... | 491·80 ,, |
| 60 feet ,, ,, | 8547 ,, .. | 427·35 ,, |

| | | Bending moment. | Total stress on boom. |
|---|---|---|---|
| 75 feet | „ „ | 6889 foot tons ... | 344·45 tons. |
| 90 feet | „ „ | 4864 „ | ... 243·20 „ |
| 105 feet | „ „ | 2470 „ | ... 123·50 „ |
| At abutment | | 0 | 0 |

In this bridge the top and bottom booms were made exactly alike. The section adopted for the boom at the centre is shown in the following sketch.




The section gives for the area:—

Gross sectional area of boom at centre...138·41 sq. ins.
Deduct for rivet holes ... ... ... ... 24·75 „

Net sectional area ... ... ... ... ...113·66 „

Hence compressive stress on top boom at centre $= \frac{574 \cdot 65}{138 \cdot 41}$ $= 4 \cdot 1518$ tons per square inch of gross area.

Tensile stress on bottom boom at centre $= \frac{574 \cdot 65}{113 \cdot 66} = 5 \cdot 0559$ tons per square inch of net area.

The sections of the booms at other points of the girder were proportioned, as far as practical conditions permitted, by the strains given in the table. It will not be necessary to give the details here.

# EXAMPLE III.

## *BOWSTRING BRIDGE FOR TWO LINES OF WAY. METHOD OF SECTIONS.*

1. When first printing these Lectures the author gave, as an example of the application of the method of sections (§ 72) to a complex example, some calculations made by him several years ago on a bowstring bridge of the type shown in Fig. 50. In ascertaining the thrust on the arched boom, in those calculations, by taking moments relatively to the joints of the bottom boom, and dividing by the length of the perpendiculars let fall from those joints on the arched boom, the moments of the stresses on the bracing bars cut by the perpendiculars, which are relatively small, were for simplicity neglected. The method would have been exact if the bowstring girder had had only a single system of triangulation (as in § 81), but in the actual case to which it was applied it was not without objection, and although the difficulty may be got over by tracing out the stresses due to each system of triangulation separately and adding them together, such a process would render a complicated calculation still more laborious. The following example has, therefore, been substituted, although in this case, as with bowstring roofs (§ 123), an arbitrary assumption has to be made as to the crossed diagonal bars, namely, that one or other of the two is absent. The error so introduced is probably not large.

2. *Preliminary assumptions and data.* The bridge is supposed to consist of two main girders with cross girders, forming a platform, attached at the joints of the main tie or

bottom boom. The vertical bars of the bracing, formed so as to act as struts, are 12 feet apart, and between the verticals are crossed diagonals of plate iron, assumed to be incapable of resisting any considerable compressive stress. The total length of the girder, taken to be the effective span, to simplify the calculations, is 204 feet ($= l$). The clear span is 194 feet. The effective depth of the girder at centre (distance from centre to centre of booms) is 21 feet ($= d$). An elevation of the girder is shown in Plate XIV., fig. 2. The assumed loads are taken at:—

| | On bridge. | On each main girder. |
|---|---|---|
| $w_1$ = total dead load per foot run | = 1·3 tons | = 0·65 ton. |
| $w_2$ = „ live „ „ | = 2·0 „ | = 1·00 „ |
| $w_1 + w_2$ | | = 1·65 „ |

3. *Maximum stresses on booms.* These are most conveniently calculated by the formulæ in § 60.

Tension on bottom boom of each main girder:—

$$= H = (w_1 + w_2)\, l^2 \div 8\,d \quad \ldots \quad \ldots \quad (1)$$
$$= 1{\cdot}65 \times 204^2 \div 8 \times 21 = 408{\cdot}7 \text{ tons.}$$

Then, for the arched boom, putting $x$ for the horizontal distance of any point from the centre of the girder, the thrust on a normal section is:—

$$H_1 = \sqrt{\{H^2 + (w_1 + w_2)\,{}^2 x^2\}} \quad \ldots \quad (2)$$
$$= \sqrt{\{167036 + 2{\cdot}7225\, x^2\}}$$

From which formula we get the following values of the thrust at various points of the arched boom:—

| Distances from centre of girder, in feet ($x$). | Thrust on arched boom with uniform load, in tons. |
|---|---|
| Centre 0 | 408·7 |
| 6 | 408·7 |
| 12 | 409·0 |
| 18 | 409·5 |
| 24 | 410·5 |
| 30 | 411·5 |
| 36 | 412·5 |
| 42 | 414·0 |

| Distances from centre of girder, in feet ($x$). | Thrust on arched boom with uniform load, in tons. |
|---|---|
| 48 ... ... ... ... | |
| 54 ... ... ... ... | 418·0 |
| 60 ... ... ... ... | 420·5 |
| 66 ... ... ... ... | 423·0 |
| 72 ... ... ... ... | 425·5 |
| 78 ... ... ... ... | 428·5 |
| 84 ... ... ... ... | 431·5 |
| 90 ... ... ... ... | 435·0 |
| 96 ... ... ... ... | 438·5 |
| Abutment 102 ... ... ... ... | 442·0 |

4. *Stresses on bracing.* The calculation of the stresses on the bracing is much more laborious in consequence of the maximum stress on each bar, arising under a different distribution of the load. We shall use the method of sections, in applying which the following steps will be necessary. Suppose the stress required on a bar cut by a vertical section through the $n$-th bay of the bottom boom, counted from the left-hand abutment, the rolling load being supposed to come on the bridge at the right abutment; then the maximum stress on that bar will arise, when all the joints to the right of the section are loaded with dead and live load, and all the joints to the left with dead load only. Assuming this distribution of load, we must calculate (§ 72):—

(*a*) The thrust ($T_n$) on the segment of the arched boom cut by the section through the bracing bar, the stress in which is required.

(*b*) The vertical component of that thrust ($T_n$ sin $i$) where $i$ is the inclination of the boom.

(*c*) The shearing force at the section ($F_n$).

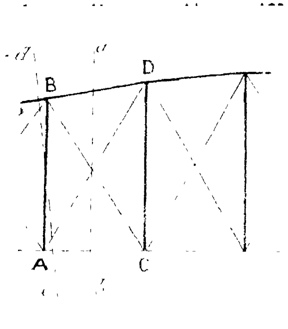
(*d*) The resultant force due to combining $T_n$ sin $i$ and $F_n$ ($F_n - T_n$ sin $i$).

(*e*) The force along the bar whose vertical component is that resultant force ($F_n - T_n$ sin $i$), Cosec $\theta$ where $\theta$ is the inclination of the bar to the horizon.

Two points, however, require consideration in the application of this method to the present case. Let Fig. 83 represent

part of the girder, and let $C$ be the $n$-th joint and $A\,C$ the $n$-th bay of the bottom boom; then for the strut and tie belonging to that bay, the maximum stress will arise when the joints from $C$ to the right abutment, inclusive, are covered by the rolling and dead load, and the remainder by the dead

FIG. 83.




load only. It remains to determine which of the two diagonals is to be supposed absent, and which of the two verticals $A\,B$, $D\,C$, belongs to the bay $A\,C$. Suppose the resultant shear on the bracing, $F_n - T_n \sin i$ at the section $a\,b$ to have been found. Then, if that shear acts upwards to the left of $a\,b$, $B\,C$ is a tie and $A\,D$ a strut. If downwards, $A\,D$ is a tie and $B\,C$ a strut. But if these diagonals are so thin as to bend when submitted to compression, the whole of that shear will act on the tie, and the strut may be supposed absent. Let $B\,C$ be the tie and $A\,D$ absent, the shear acting upwards to the left of a vertical section in the bay $A\,C$. Then the section through $A\,B$ must be taken in the direction $c\,d$, nearly vertical, but to the left of $B$ and to the right of $A$, that section if $A\,D$ is absent cutting only three bars, and permitting the stress on $A\,B$, which will be found to be compressive, to be determined.

In calculating the resultant shear on $a\,b$, the vertical component of the thrust on the $n$-th bay of the arched boom ($B\,D$) is combined with the shearing force between the ($n$-1)th and $n$-th joints of the bottom boom. In calculating the resultant shear on $c\,d$, the vertical component of the thrust on the ($n$-1)th bay of the arched boom is combined with the

shearing force between the same joints. The rolling load in both cases extending from the $n$-th joint, $C$, to the right abutment.

5. *Calculation of supporting force.* The supporting force at the left abutment, when the rolling load extends from $n$-th joint of the bottom boom to the right abutment, may be found as follows:—Let $N$ be the number of bays into which the bottom boom is divided by the bracing (so that in this case $N = 17$). Then with the assumed distribution of load $n - 1$ joints next the left abutment carry dead load only, and $N - n$ carry dead and live load. Let $P_n$ be the supporting force at the left abutment, with this distribution of load.

The weight of rolling load on the girder, excluding the part supported directly over the right abutment, is

$$= (N - n)\, l\, w_2 \div N$$

The distance of the centre of gravity of that load from the right abutment is

$$= (N - n + 1)\, l \div 2N$$

and the reaction at the left abutment due to this live load is (§ 5)

$$= \frac{(N - n)\,(N - n + 1)\, l\, w_2}{2\, N^2}$$

adding the reaction due to the dead load, or half the total dead load, exclusive of that supported directly at the abutments,

$$P_n = \frac{(N - 1)\, l\, w_1}{2\, N} + \frac{(N - n)\,(N - n + 1)\, l\, w_2}{2\, N^2} \ldots\ldots (3.)$$

Substituting $n = 1, 2, 3 \ldots 17$ successively, we get the following values of the supporting force for the successive positions of the rolling load:—

| | Supporting force at left abutment in tons. | | |
|---|---|---|---|
| $n =$ | Due to dead load. | Due to live load. | Total ($P_n$) |
| 1 | 62·4 | 96·0 | 158·4 |
| 2 | ,, | 84·7 | 147·1 |
| 3 | ,, | 74·1 | 136·5 |
| 4 | ,, | 64·2 | 126·6 |

| $n =$ | Supporting force at left abutment in tons. Due to dead load. | Due to live load. | Total $(P_n)$ |
|---|---|---|---|
| 5 | 62·4 | 55·0 | 117·4 |
| 6 | ,, | 46·5 | 108·9 |
| 7 | ,, | 38·8 | 101·2 |
| 8 | ,, | 31·8 | 94·2 |
| 9 | ,, | 25·4 | 87·8 |
| 10 | ,, | 19·8 | 82·2 |
| 11 | ,, | 14·7 | 77·1 |
| 12 | ,, | 10·6 | 73·0 |
| 13 | ,, | 7·1 | 69·5 |
| 14 | ,, | 4·2 | 66·6 |
| 15 | ,, | 2·1 | 64·5 |
| 16 | ,, | 0·7 | 63·1 |
| 17 | ,, | 0 | 62·4 |

6. *Calculation of thrust on arched rib with partial loads.* We require the thrust on the $n$-th and the $(n-1)$-th bay of the arched rib, when the rolling load extends from the $n$-th joint of the bottom boom to the right abutment. That thrust is obtained, by dividing the bending moment at the joint of the bottom boom perpendicularly opposite the segment under consideration, by the length of the perpendicular from the joint on the segment. From the left abutment to the centre the $n$-th joint of the bottom boom is opposite the $n$-th bay of the arched boom. From the centre to the right abutment the $(n-1)$-th joint of the bottom boom is opposite the $n$-th bay of the arched boom (Plate XIV.) We require, therefore, in determining the thrust on the $n$-th segment of the arched boom, the bending moment at the $n$-th joint from the left abutment to the centre, and that at the $(n-1)$-th joint for the remainder. For the thrust on the $(n-1)$-th bay of the arched boom, we require the bending moment at the $(n-1)$-th joint of the bottom boom from the left abutment to the centre, and that at the $(n-2)$-th joint beyond the centre.

The bending moment at the $n$-th joint when the rolling load covers the bridge from that joint to the right abutment is, considering the forces acting to the left of the joint,

$$M_n = \frac{P_n\, l\, n}{N} - \frac{w_1\, l^2}{N^2} \left\{ 1 + 2 + \ldots\ldots + (n-1) \right\} \ldots\ldots (4.)$$

That at the $(n-1)$-th joint in the same case is

$$M_{n-1} = \frac{P_n\, l\,(n-1)}{N} - \frac{w_1\, l^2}{N^2} \left\{ 1 + 2 + \ldots\ldots (n-2) \right\} \ldots\ldots (5.)$$

That at the $(n-2)$-th joint is

$$M_{n-2} = \frac{P_n\, l\,(n-2)}{N} - \frac{w_1\, l^2}{N_2} \left\{ 1 + 2 + \ldots\ldots (n-3) \right\} \ldots\ldots (6.)$$

The first factor on the right of these equations is the moment of the reaction at the left abutment, and the second the sum of the moments of the loads at the joints between that abutment and the joint at which the moment is taken (§ 8).

Bending moments in foot tons.

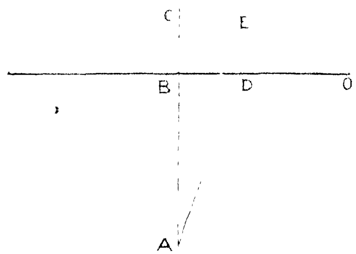
| $n =$ | $M_n$ | $M_{n-1}$ | $M_{n-2}$ |
|---|---|---|---|
| 1 | 1901 | 0 | ... |
| 2 | 3434 | 1764 | ... |
| 3 | 4633 | 3182 | ... |
| 4 | 5515 | 4276 | ... |
| 5 | 6108 | 5073 | ... |
| 6 | 6437 | 5598 | ... |
| 7 | 6536 | 5882 | ... |
| 8 | 6422 | 5947 | ... |
| 9 | ... | 5808 | 5410 |
| 10 | ... | 5508 | 5270 |
| 11 | ... | 5030 | 4957 |
| 12 | ... | 4488 | 4548 |
| 13 | ... | 3831 | 4026 |
| 14 | ... | 3089 | 3413 |
| 15 | ... | 2319 | 2762 |
| 16 | ... | 1530 | 2085 |
| 17 | ... | 748 | 1404 |

To obtain from these moments the thrust, on the segments of the top boom perpendicularly opposite the joints of the bottom boom at which the moments have been taken, we must know the lengths of the perpendiculars let fall from those joints on the top boom. These may be readily measured

from the drawing with accuracy enough for practical purposes. But they may be calculated in the following manner:—

Let $O D$ represent $n$ divisions, and $O B = \frac{1}{2} N$ divisions of the bottom boom, then if $A$ is the centre of the curve of the

FIG. 84.




top boom, $D E$ is the perpendicular from the $n$-th joint on the top boom. Let $r = A C = A E =$ radius of boom.

$$p_n = D E = r - \sqrt{\{D B^2 - (r - B C)^2\}} \ldots\ldots (7.)$$

But if $d = B C$, the versed sine of the curve,

$$r = \frac{l^2 + 4 d^2}{8 d}$$

$$B D = \frac{l}{2} - \frac{l n}{N}$$

In the present case,

$$r = (204^2 + 4 \times 21^2) \div (8 \times 21) = 258{\cdot}21 \text{ feet.}$$

$$D E = 258{\cdot}21 - \sqrt{(10404 - 1224 n + 36 n^2 + 56269)}$$
$$= 258{\cdot}21 - 6 \sqrt{(1852 - 34 n + n^2)}$$

| $n =$ | Length of perpendicular from $n$-th joint on arched boom in feet. ($p_n$) |
|---|---|
| 1 or 16 | 4·53 |
| 2 ,, 15 | 8·49 |
| 3 ,, 14 | 11·97 |
| 4 ,, 13 | 14·91 |
| 5 ,, 12 | 17·31 |
| 6 ,, 11 | 19·17 |
| 7 ,, 10 | 20·31 |
| 8 ,, 9 | 20·91 |

Dividing the moments in preceding table by the corresponding perpendiculars in this table, we get the following values:—

$$T_n = M_n \div p_n \text{ ........... } (8.)$$

| $n =$ | Thrust on arched boom in tons. On $(n-1)$-th segment. | On $n$-th segment. |
|---|---|---|
| 1 | 0 | 419·6 |
| 2 | 389·5 | 404·5 |
| 3 | 374·5 | 387·1 |
| 4 | 357·2 | 369·9 |
| 5 | 340·2 | 352·9 |
| 6 | 323·4 | 335·8 |
| 7 | 306·8 | 321·8 |
| 8 | 292·8 | 307·1 |
| 9 | 277·7 | ... |
| 10 | ... | 263·4 |
| 11 | 237·0 | 247·7 |
| 12 | 224·0 | 234·1 |
| 13 | 210·0 | 221·3 |
| 14 | 197·1 | 207·2 |
| 15 | 185·2 | 193·7 |
| 16 | 174·2 | 180·2 |
| 17 | 165·4 | 165·1 |

Lastly, to complete this step of the calculation, we must ascertain the vertical components of these thrusts. It is easy to show that the inclination of the curve at $E$ (Fig. 84) to the horizon is—

$$i = \tan^{-1} \frac{D\,B}{A\,B} = \tan^{-1} \frac{\frac{l}{2} - \frac{n\,l}{N}}{r - d}$$

Or in the case under consideration,

$$\tan i = \frac{102 - 12n}{258{\cdot}21 - 21} = \frac{17 - 2n}{39{\cdot}535}$$

If $T_n$ be the thrust on a normal section at $E$, opposite the $n$-th joint, then the vertical component of that thrust is $=$ $T_n \sin i$, and

$$\sin i = \frac{1}{\sqrt{(1 + \cot^2 i)}} = \frac{1}{\sqrt{\left\{1 + \left(\frac{39 \cdot 535}{17 - 2n}\right)^2\right\}}}$$

$$= \frac{17 - 2n}{\sqrt{(4n^2 - 68n + 1852)}} \ldots\ldots(9).$$

This formula gives the following values for the sine of the inclination of the arched boom at the points perpendicularly opposite the joints of the bottom boom:—

| Bay of arched boom. | Sin $i$ = | Bay of arched boom. | Sin $i$ = |
|---|---|---|---|
| 1 or 17 | ·3548 | 5 or 13 | ·1743 |
| 2 „ 16 | ·3123 | 6 „ 12 | ·1254 |
| 3 „ 15 | ·2681 | 7 „ 11 | ·0757 |
| 4 „ 14 | ·2220 | 8 „ 10 | ·0253 |

Multiplying the thrusts by these values, we get for the vertical components of the thrusts on the arched boom in each bay of the bridge the following values:—

| $n$ = | Vertical component of thrust on arched booms, in tons. On $(n-1)$-th segment. | On $n$-th segment. |
|---|---|---|
| 1 | 0 | 148·9 |
| 2 | 138·2 | 126·3 |
| 3 | 117·0 | 103·8 |
| 4 | 95·7 | 82·1 |
| 5 | 75·5 | 61·5 |
| 6 | 56·4 | 42·1 |
| 7 | 38·5 | 24·3 |
| 8 | 21·2 | 7·8 |
| 9 | 7·0 | 0 |
| 10 | 0 | 6·7 |
| 11 | 6·0 | 18·7 |
| 12 | 16·9 | 29·4 |
| 13 | 26·3 | 38·6 |
| 14 | 34·4 | 46·0 |
| 15 | 41·1 | 52·0 |
| 16 | 46·7 | 56·3 |
| 17 | 51·6 | 58·6 |

7. *Shearing force at each bay of bridge, with partial loads.* The shearing force on a vertical section through the $n$-th bay of bottom boom, when the rolling load extends from the $n$-th

joint to the right abutment, will be, considering the force, to the left of the section—

$$F_n = P_n - \frac{w_1 l}{N} (n - 1) \ldots\ldots\ldots (10.)$$

The shearing force acts upwards to the left of the section if $F_n$ is positive, and downwards if negative.

| Bay of bottom boom. $n =$ | Shearing force, in tons. ($F_n$) | Bay of bottom boom. $n =$ | Shearing force, in tons. ($F_n$) |
|---|---|---|---|
| 1 | 158·4 | 9 | 25·4 |
| 2 | 139·3 | 10 | 12·0 |
| 3 | 120·9 | 11 | − 0·9 |
| 4 | 103·2 | 12 | − 12·8 |
| 5 | 86·2 | 13 | − 24·1 |
| 6 | 69·9 | 14 | − 34·8 |
| 7 | 54·4 | 15 | − 44·7 |
| 8 | 39·6 | 16 | − 53·9 |
| | | 17 | − 62·4 |

8. *Stresses on bracing bars in compression.* The shear on any bar is found by combining $F_n$ and the vertical component of the thrust and multiplying the resultant by the cosecant of the inclination of the bar to the horizon. For the vertical bars, if the shear is positive on left of section, the shearing force of $n$-th bay of bottom boom is to be combined with the vertical component of thrust of ($n$-1)th bay of rib, and the vertical bar belonging to the $n$-th bay is that at the ($n$-1)th joint.

| $n =$ | Shearing force. | Thrust. | Shear on bar at ($n$-1)th joint, in tons. |
|---|---|---|---|
| 2 | 139·3 | 138·2 | 1·1 |
| 3 | 120·9 | 117·0 | 3·9 |
| 4 | 103·2 | 95·7 | 7·5 |
| 5 | 86·2 | 75·5 | 10·7 |
| 6 | 69·9 | 56·4 | 13·5 |
| 7 | 54·4 | 38·5 | 15·9 |
| 8 | 39·6 | 21·2 | 18·4 |
| 9 | 25·4 | 7·0 | 18·4 |
| 10 | 12·0 | 0 | 12·0 |
| 11 | — 0·9 | — 6·0 | 5·1 |
| 12 | — 12·8 | — 16·9 | 4·1 |
| 13 | — 24·1 | — 26·3 | 2·2 |

As the bars are vertical, the shear on the bars is the stress on the bars. It is unnecessary to carry the calculation beyond the point at which the shear changes sign.

9. *Stresses on diagonal bracing bars, in tension.* When the shear is positive on the left of a section, the diagonal sloping from the ($n$-1)th joint of arched to $n$-th joint of bottom boom is in tension. The vertical component of thrust on $n$-th bay of arched boom has to be combined with the shearing force in $n$-th bay.

| $n$ = | Shearing force. | Thrust. | Shear on diagonal bar at $n$-th bay, in tons. |
|---|---|---|---|
| 2 | 139·3 | 126·3 | 13·0 |
| 3 | 120·9 | 103·8 | 17·1 |
| 4 | 103·2 | 82·1 | 21·1 |
| 5 | 86·2 | 61·5 | 24·7 |
| 6 | 69·9 | 42·1 | 27·8 |
| 7 | 54·4 | 24·3 | 30·1 |
| 8 | 39·6 | 7·8 | 31·8 |
| 9 | 25·4 | 0 | 25·4 |
| 10 | 12·0 | — 6·7 | 18·7 |
| 11 | — 0·9 | — 18·7 | 17·8 |
| 12 | — 12·8 | — 29·4 | 16·6 |
| 13 | — 24·1 | — 38·6 | 14·5 |
| 14 | — 34·8 | — 46·0 | 11·2 |

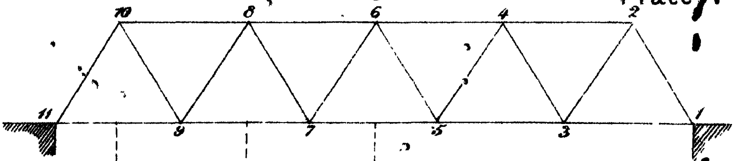
The angles of inclination of the bars may be measured off a drawing, and the stress obtained by multiplying the shear on the bar by the cosecant of the corresponding angle.

10. The stresses thus obtained are due to a rolling load coming on the bridge at the right abutment. By writing them against the corresponding bars in the other half of the bridge, the stresses due to a rolling load arriving at the left abutment will be found. Where two stresses come to be written against one bar, the bar must, of course, be proportioned to the greater.

It will not always be necessary to calculate the stresses for every bay as has here been done. The process is equally applicable to finding the stresses in a single bay.

Plate V

Fig. 1



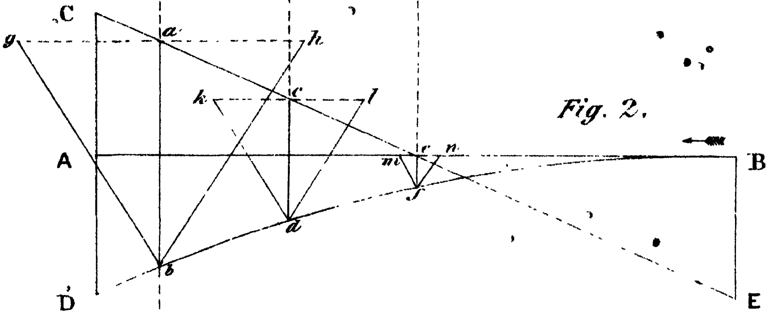




Fig. 2.

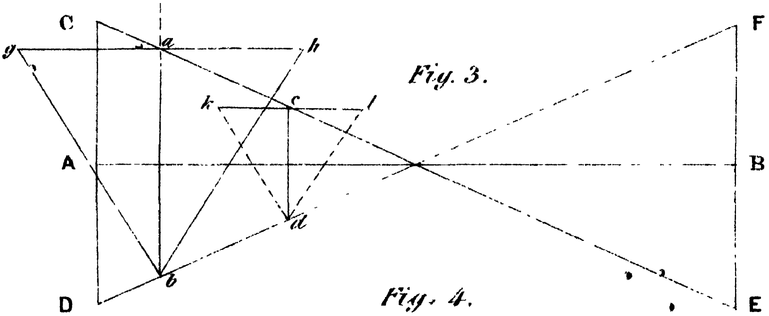
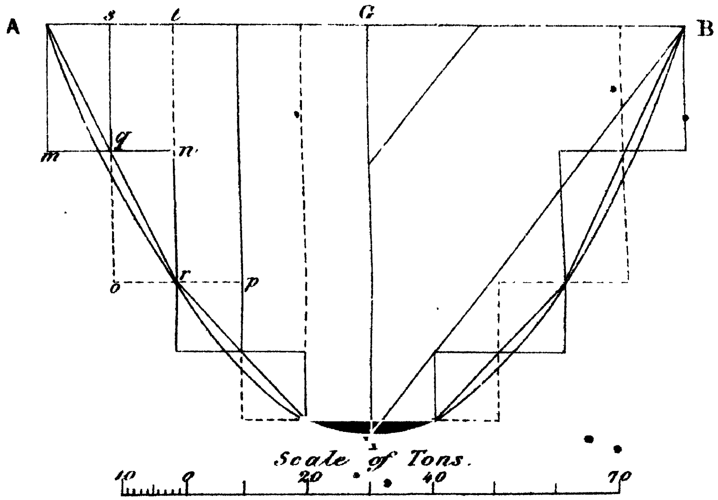



Fig. 3.

Fig. 4.



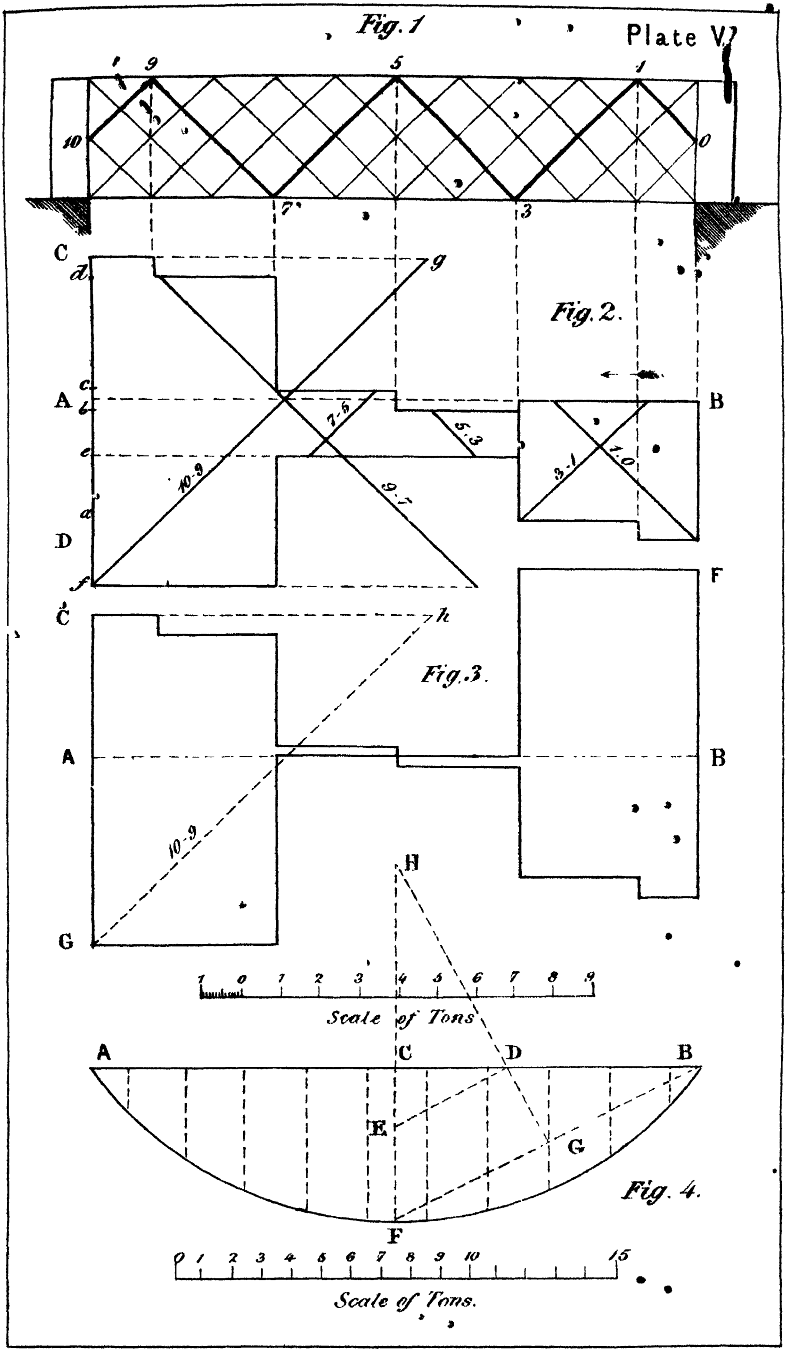


Plate V
Fig. 1
9
5
1
10
0
7
3
C
d
g
Fig. 2.
A
c
b
B
7·5
5·3
e
10·9
9·7
3·1
1·0
a
D
f
F
C
h
Fig. 3.
A
B
10·9
G
H
1 0 1 2 3 4 5 6 7 8 9
Scale of Tons
A
C
D
B
E
G
Fig. 4.
F
0 1 2 3 4 5 6 7 8 9 10 15
Scale of Tons.

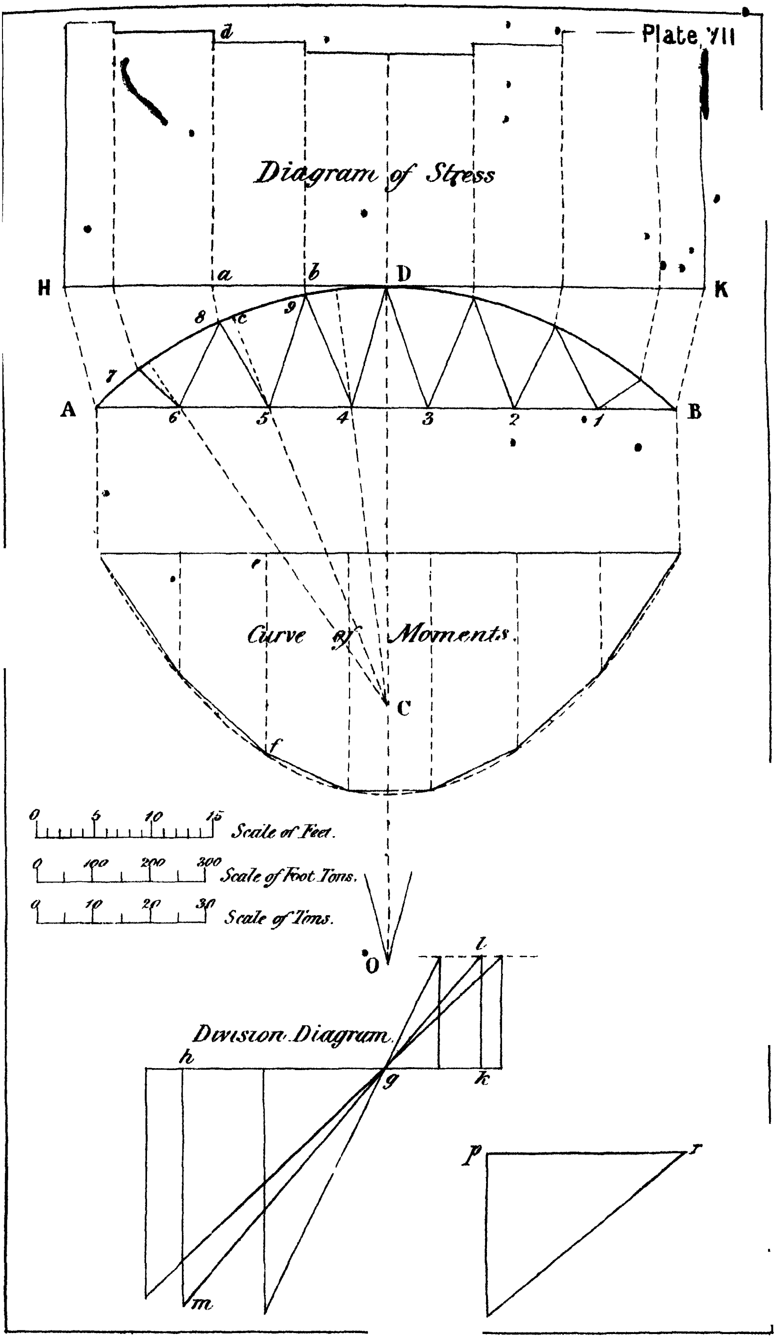


Plate VII
Diagram of Stress
Curve of Moments.
Division Diagram.
Scale of Feet.
Scale of Foot Tons.
Scale of Tons.
H
K
A
B
D
C
O

Plate VIII

*Fig. 1*

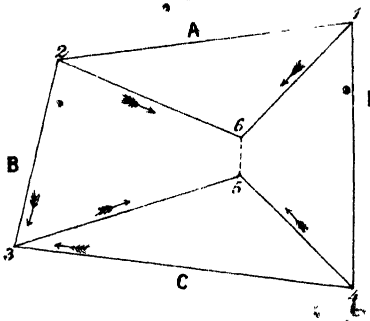



*Fig. 2*

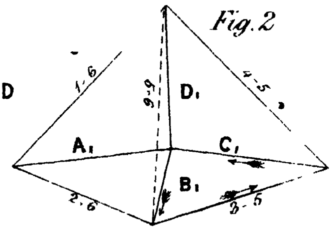



*Fig. 3*

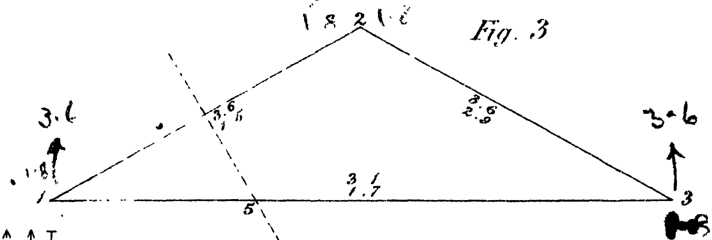



*Fig. 4*

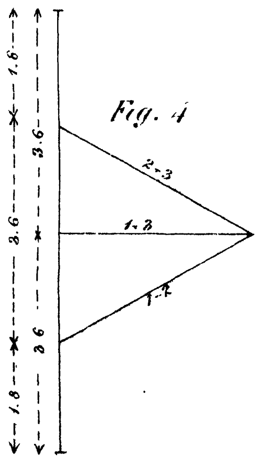
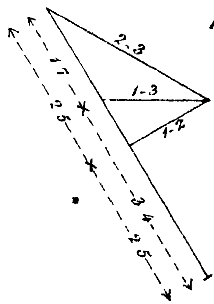



*Fig. 5.*




*Scales.*

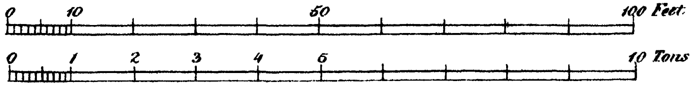
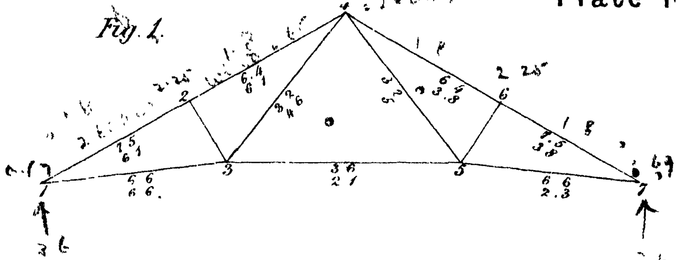


Plate II

*Fig. 1.*



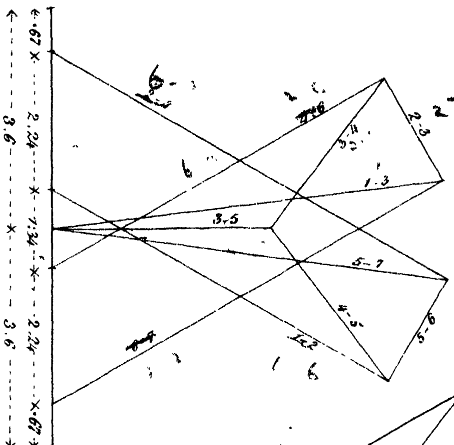


*Fig. 2.*

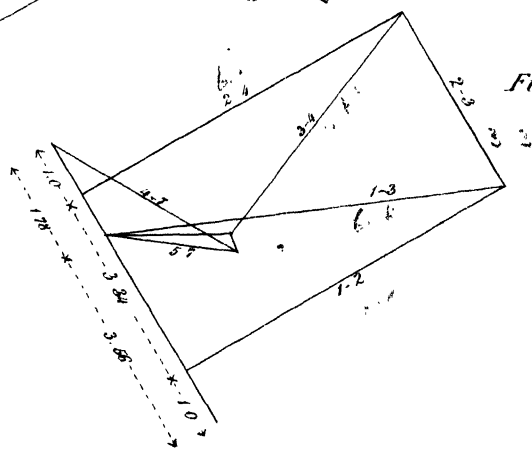


*Fig. 3*

*Scales*

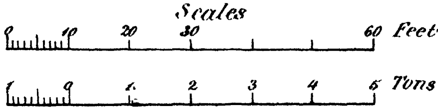
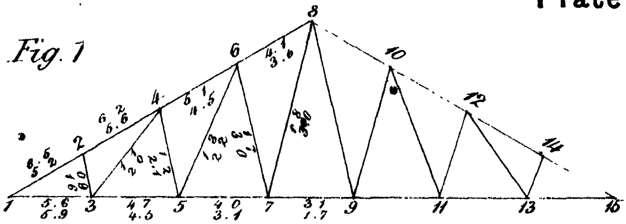


Plate X

*Fig. 1*



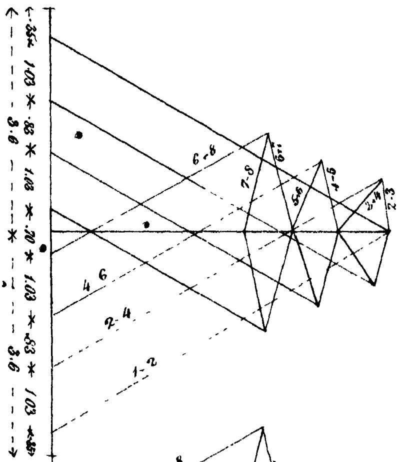




*Fig 2.*

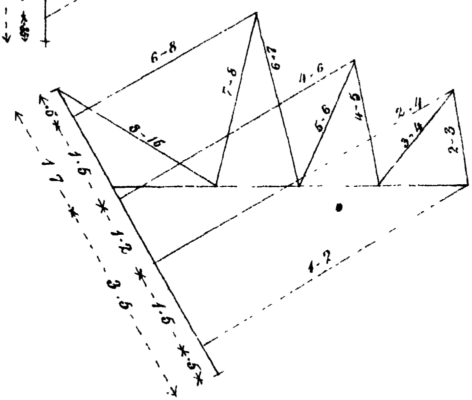
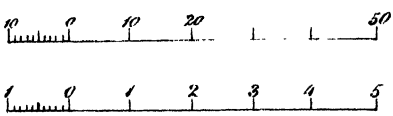



*Fig. 3*

*Scales*




*Feet*

*Tons.*

Plate XI

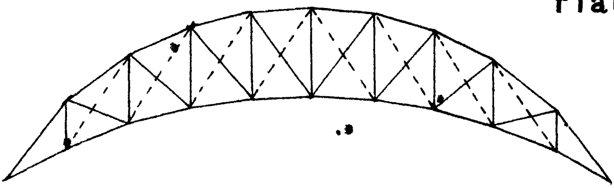


*Fig. 1*

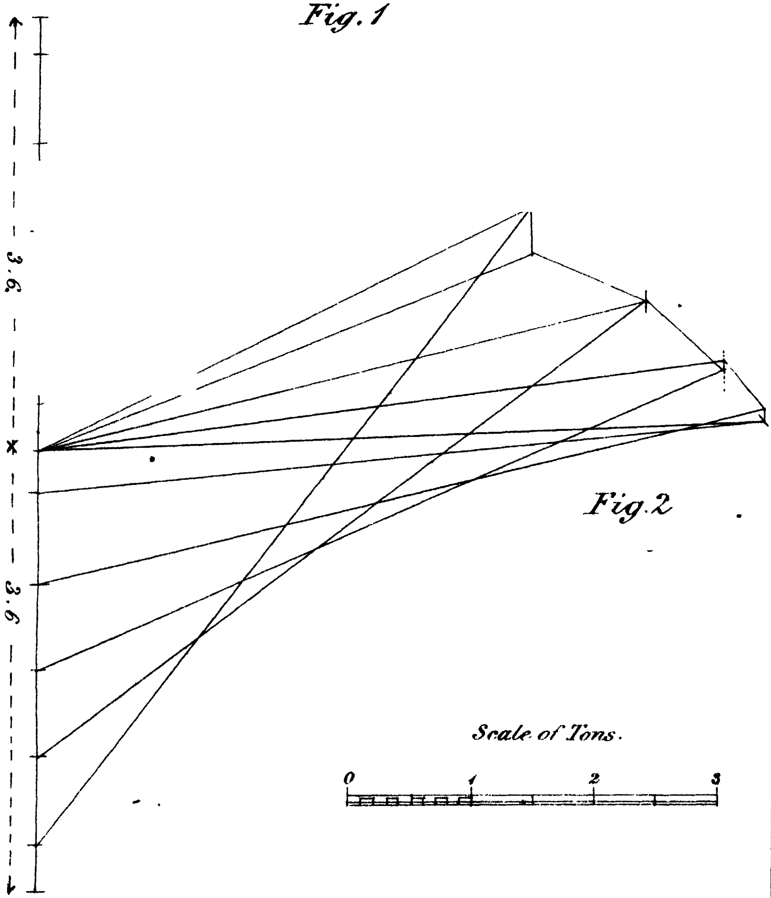



*Fig. 2*

Plat II

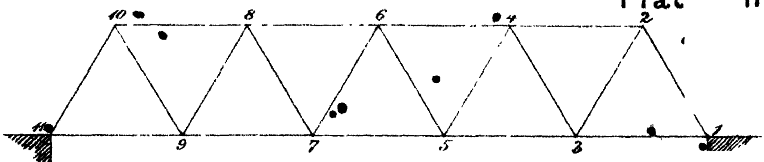



Fig. 1.

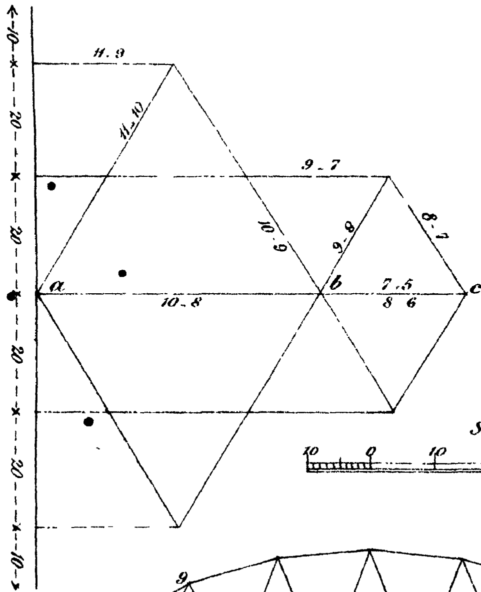
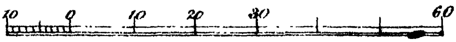



Fig. 2

Scale of Tons



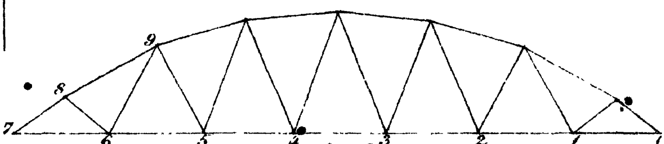




Fig. 3.

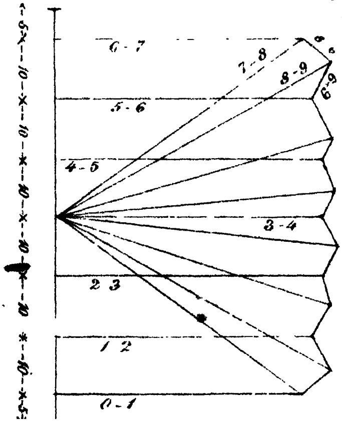



Fig. 4.

Scale of Tons

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